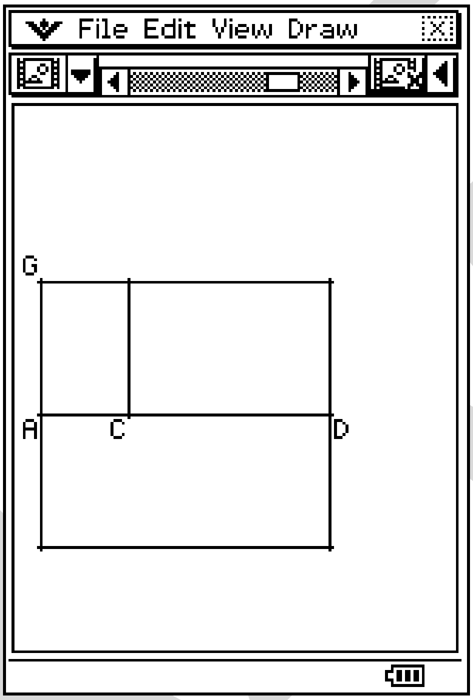
Seeing Double



Looking closely at algebraic identities.

Prince Alfred College
Founded 1869

Noel Baker Centre
for School Mathematics

CASIO®

Seeing Double

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Section *Page*

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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Legend.

EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just *explore and think!*

At certain points the learning process should have generated some ***burning mathematical questions*** that should be discussed and pondered, and then answered as you learn more!

Time to Formalise.

These notes document the learning that has occurred up to this point,
using a degree of formal mathematical language and notation.

Examples.

Illustrations of the mathematics at hand, used to answer questions.

1. Look hard … what do you see?

Mathematical notation can be used to describe the behaviour of things that we see.

If used well it is a simple and powerful tool.

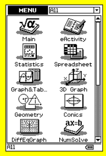
At best, behaviour can be fully captured by a few symbols – the fewer the better!

1.1 EAT 1

Open the *ClassPad 300* geometry file **`dista1`** (in the **`distrib`** folder) and run the animation that has been added.

Your task is to describe / 'name' the thing(s) you see.
It may help you to think about **what changes** and **what stays the same**.

Record your descriptions.
Compare and discuss these with your class.



9.1

1.2 EAT 2

Repeat this for **`dista2`** and **`dista3`**.
Record and discuss the *different* **ways that what you see can be described.**

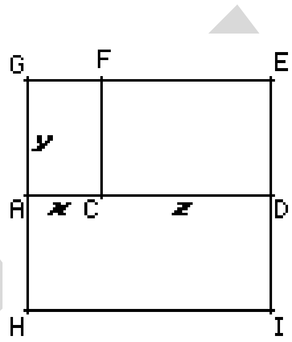
1.3 EAT 3

Explore and think in the same way about the sequence of animated geometry files **`distb1`**, **`distb2`** and **`distb3`**.

Explore and think in the same way about the sequence of animated geometry files **`distc1`**, **`distc2`** and **`distc3`**.

One group who looked at the previous animations labelled the diagram (left) and saw

$$y \times (x + z)$$
$$xy + yz$$



Can you see what they are describing?
Can you see how they chose to describe it?

Another group saw

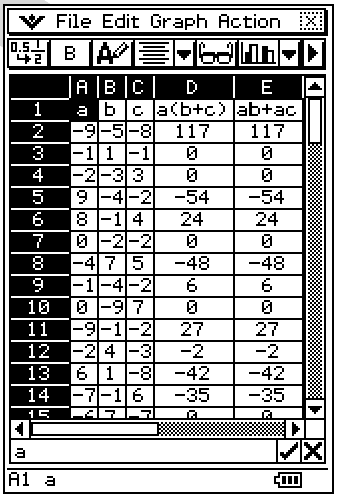
$$2y \times (x + z)$$
$$2(xy + yz)$$
$$xy + yz + xy + yz$$
$$2xy + 2yz$$
$$xy + yz + y \times (x + z)$$

Can you see what they were describing?
Can you see how they chose to describe it?

These algebraic expressions can be thought of as *non-identical mathematical twins*

They don't *look* the same but they are *essentially* the same, equivalent in value and in a way, interchangeable[1]

This 'equivalence' means that the expressions take the same value when values for the variables are assigned.



	A	B	C	D	E
1	a	b	c	a(b+c)	ab+ac
2	−9	−5	−8	117	117
3	−1	1	−1	0	0
4	−2	−3	3	0	0
5	9	−4	−2	−54	−54
6	8	−1	4	24	24
7	0	−2	−2	0	0
8	−4	7	5	−48	−48
9	−1	−4	−2	6	6
10	0	−9	7	0	0
11	−9	−1	−2	27	27
12	−2	4	−3	−2	−2
13	6	1	−8	−42	−42
14	−7	−1	6	−35	−35

Open the *ClassPad 300* spreadsheet file `distrib` (in the `distrib` folder) to observe 'equivalence' in action.

This spreadsheet focuses on the twins described by the first group above.

If we are sure that these expressions return the same value for **all** possible values that the variables could take (a spreadsheet with infinite rows perhaps?) then we can say that they are *equivalent* and they can be linked by an = sign i.e. …

[1] Unlike human non-identical twins, who are unique individuals in their own right and are in no way interchangeable!

$$a(b+c) = ab + ac$$

Or to put it another way

$$ab + ac = a(b+c)$$

This is commonly called the **Distributive Law**

Note: Our demonstrations do not prove the equivalence needed to make this a mathematical *law*, but others have provided this proof, establishing it as law!

1.4 Can you use your knowledge – 1?

1. Write down 6 sets of non-identical mathematical twins that illustrate the Distributive Law.

2. Write each of the following as a product of two factors

1.	$6x+2$	5.	$10x-25$	9.	$-6x+9$
2.	$4x+12$	6.	$2x+5$	10.	$32-8x$
3.	$10x+20$	7.	$9x-4$	11.	$-5-15x$
4.	$9x-12$	8.	$-2x-6$		

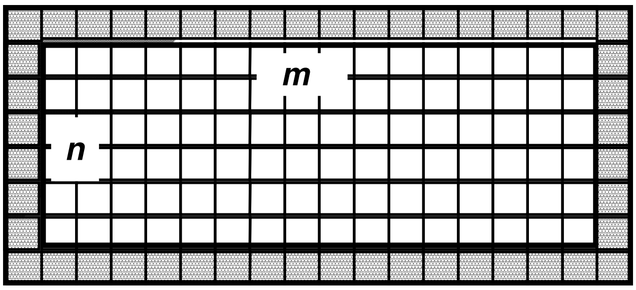
3. Write each of the following as the sum of two elements

1.	$2(x+y)$	5.	$-4(x-5)$	9.	$3x^2(2x-3y)$
2.	$8(2x-3)$	6.	$2x(x+2y)$	10.	$ab(a+2b)$
3.	$x(y+5)$	7.	$-x(x-6)$	11.	$-5y^3(9x^2-y)$
4.	$x(x+2)$	8.	$5x^2(y-1)$		

4. Picture this – a nicely tiled, rectangular swimming pool with a tiled border which is one tile wide.

If the pool is *m* tiles long and *n* tiles wide. How many border tiles are there?

Derive as many expressions for the number of border tiles as you can



5. Write down one or more non-identical mathematical twins for each of the following expressions

1.	$2x^2 - 8x$	6.	$3x(x - y + 3)$	11.	$10 - x(x - 3)$
2.	$4x(2x + 5y)$	7.	$x(2x + 3z - 5)$	12.	$2(x - 5) + x(y + 3)$
3.	$10 - 18x^2$	8.	$12y + 4y^2$	13.	$x(x - 4) + 7(x - 4)$
4.	$3yz - 24xz$	9.	$ab(a + b)$	14.	$3(1 - z) + y(1 - z)$
5.	$18x^2 - 23x^2 y$	10.	$x(x + 7) - 4x$	15.	$z(x - 4) - 2z(4 - x)$

6. **Research** the mathematical terms *expand* and *factorise*.

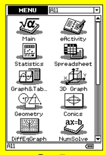
Explain their meanings in your own words (including examples as required).

2. Looking at squares and other beasts.

2.1 EAT 4

Open the sequence of *ClassPad 300* geometry files `psquara1 to psquara4` (in the `psquare` folder) and run the animations that have been added.
As before, your task is to describe / 'name' the thing(s) you see.

Record your descriptions.
Compare and discuss these with your class.



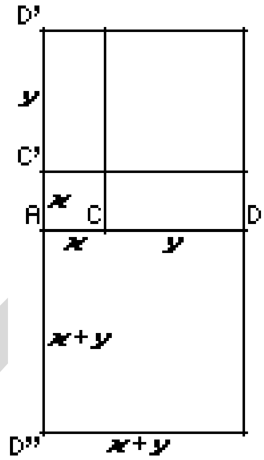
9.1

2.2 EAT 5

Repeat this for `psquarb1` to `psquarb4`.

Summarise your findings from these activities.

A ‘snapshot’ of `psquarb4` is shown here, with some labels added. One group saw



$$x^2 + xy + xy + y^2 \quad \text{or}$$
$$x^2 + 2xy + y^2$$

Can you see what they are describing?
Can you see how they chose to describe it?

Another group saw

$$(x + y)^2$$

Can you see what they are describing?
Can you see how they chose to describe it?

This implies that

$$x^2 + 2xy + y^2 = (x + y)^2$$

This is known as the **Perfect Square Identity.**

But why doesn’t $(x + y)^2 = x^2 + y^2$?

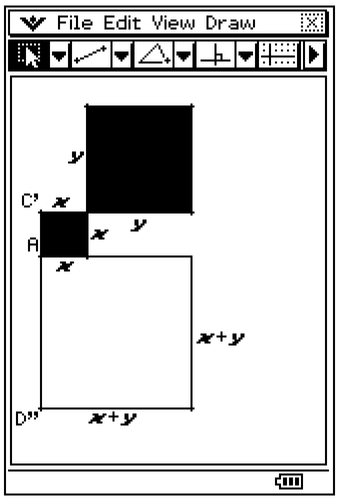
This common misunderstanding seems to follow from the correct application of the Distributive Law that says $2(x + y) = 2x + 2y$.

As it turns out, however, you can’t “distribute” a “squared” like you can a “times 2”

If $(x + y)^2 = x^2 + y^2$ then the large white square would have the same area as the two dark squares added.

That clearly is not the case.

There are two rectangles ‘missing’.
Each has an area of xy, so there is $2xy$ ‘missing’.



Another way to see this is through a spreadsheet.
Open the *ClassPad 300* spreadsheet file `psquare2` (in the `psquare` folder).

	A	B	C	D	E	F	G	H
1	a	b	a^2	b^2	a^2+b^2	(a+b)^2	2ab	a^2+2ab+l
2	−1	8	1	64	65	49	−16	49
3	−8	−6	64	36	100	196	96	196
4	4	4	16	16	32	64	32	64
5	5	−4	25	16	41	1	−40	1
6	−7	1	49	1	50	36	−14	36
7	4	−2	16	4	20	4	−16	4
8	9	7	81	49	130	256	126	256
9	4	−2	16	4	20	4	−16	4
10	−3	−5	9	25	34	64	30	64
11	−9	9	81	81	162	0	−162	0
12	8	3	64	9	73	121	48	121
13	8	8	64	64	128	256	128	256
14	−5	−4	25	16	41	81	40	81
15	−8	9	64	81	145	1	−144	1

You can see that $a^2 + b^2$ (column E) is not equal to $(a + b)^2$ (column F) and that what is ‘missing’ is $2ab$ (column G)

2.3 Can you use your knowledge – 2?

1. Write down 6 examples of the Perfect Square Identity – include a minus sign in at least 2 of them.

2. Write each of the following as the sum of three terms.

1.	$(x+7)^2$	5.	$(x-5)^2$	9.	$(8-w)^2$
2.	$(y+3)^2$	6.	$(2x+1)^2$	10.	$(5i+2j)^2$
3.	$(x+1)^2$	7.	$(y-6)^2$	11.	$(4a-3b)^2$
4.	$(z-10)^2$	8.	$(5x+4)^2$	12.	$(6x-11y)^2$

3. Which of the following are examples of the *Perfect Square Identity*?

1.	$x^2-14x+49$	4.	$(72x-13y)^2$	7.	$x^2-12x-36$
2.	y^2+9	5.	$y^2-25x+10$	8.	$a^2+16b+64$
3.	$z^2+4x+16$	6.	$121-22z+z^2$	9.	$9i^2+12ij^2+4j^4$

4. Add the missing term to make the following *Perfect Squares*.

1.	$x^2+\_\_+4$	4.	$a^2-12a+\_\_$	7.	$16x^2+\_\_+9y^2$
2.	$x^2-\_\_+100$	5.	$\_\_-10x+25$	8.	$25w^2-60wx+\_\_$
3.	$y^2+18y+\_\_$	6.	$4k^2+\_\_+36$	9.	$p^4+\_\_+49q^2$

5. Change a term to make these *Perfect Squares*

1.	$x^2-10x-25$	3.	$a-2ab+b^2$	5.	$25p^2+50pq+1$
2.	$y^2+9x+81$	4.	$3k^2+12km+4m^2$	6.	$16x^2-48xy+36y^4$

6. For which parts of question 5 is there more than one correct answer?

   For those parts, provide an additional answer.

7. Where possible, write the following in the form $(a+b)^2$

1.	$y^2+20y+100$	5.	$x^2+2xy+y^2$	9.	$9x^2-24xy+16y^2$
2.	$x^2-8x+16$	6.	$a^2-12a-144$	10.	$25p^2+80pq+64q^2$
3.	k^2-6k+9	7.	$1-2z-z^2$	11.	$x^4-18x^2y+81y^2$
4.	$25+10m+m^2$	8.	$4a^2+24a+36$	12.	$49a^2-42ab+36b^2$

3. And now speaking generally…

$(a+b)^2$, or its non-identical twin $(a+b)(a+b)$, is an example of a process sometimes called *binomial expansion*.

Roughly translated *binomial* means *two number*, meaning that this is an example of

a factor with two terms multiplied by another factor with two terms.

What we have learned to far covered only the case where the two factors are identical (hence the 'squared'). What about binomial expansion in general?

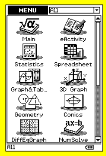
3.1 EAT 6

Consider the general binomial expansion $(a+b)(c+d)$.

What sort of construction or shape would be associated with this expression?

What does this suggest about *a non-identical twin expression* for $(a+b)(c+d)$?

Check that any non-identical twin does in fact return the same value as $(a+b)(c+d)$ for range of different inputs.



9.2

Can you prove that these expressions are equivalent, using only the identities studied so far in the topic?

3.2 Can you use your knowledge – 3?

1. Write down 6 sets of non-identical mathematical twins where one is of the form $(a+b)(c+d)$.

2. Write each of the following as the sum of four terms, and then collect *like terms* where possible.

1.	$(x+5)(x+2)$	5.	$(a+3)(b-4)$	9.	$(5x+4)(3y-2z)$
2.	$(x-6)(y-7)$	6.	$(2x-3)(x-9)$	10.	$(x-6)(x+6)$
3.	$(x+1)(x-15)$	7.	$(15x-1)(10x+1)$	11.	$(1+z)(1-z)$
4.	$(x+p)(x+q)$	8.	$(3y+4)(2y+1)$	12.	$(4a-3b)(4a+3b)$

3. Look at Question 2 parts 1 to 4 (above).

   These "twins" with three terms are called *quadratic trinomials* as they contain *three terms*, a quadratic term containing x^2, a linear term containing x and a constant term (independent of x).

   a. Explain how the coefficient of linear term can be obtained.
   b. Explain how the constant term can be obtained.

4. Use this "sum and product" property to write the following as the product of two binomial factors.

1.	x^2+8x+7	5.	x^2-3x-4	9.	$x^2+11x+24$
2.	$x^2+10x+16$	6.	$x^2-11x+28$	10.	x^2-6x+8
3.	$x^2+7x+12$	7.	x^2-x-20	11.	$x^2-10x+9$
4.	$x^2+3x-10$	8.	$x^2-9x+18$	12.	$x^2-9x-36$

5. In a similar way, write the following as the product of two binomial factors.

1.	$2x^2+7x+3$	5.	$4x^2-8x-5$	9.	$4x^2+33x+8$
2.	$3x^2+8x+5$	6.	$6x^2+11x-10$	10.	$12x^2+4x-21$
3.	$2x^2+7x+6$	7.	$10x^2+3x-7$	11.	$10x^2+x-24$
4.	$5x^2-12x+4$	8.	$6x^2-23x+15$	12.	$18x^2-33x+14$

This process is sometimes called **binomial expansion** and is summarised by the identity

$$(a+b)(c+d)=ac+ad+bc+bd$$

When dealing with quadratic trinomials it is sometimes written as

$$(x+p)(x+q)=x^2+(p+q)x+pq$$

4. Another interesting case…

Look closely at Question 2 parts 10, 11 and 12 on the previous page. You may notice that their non-identical twins differ from the twins in the other parts of this question.

Discuss with your classmates the way that these twins differ from the others that you have seen.

Discuss what causes this difference to come about.

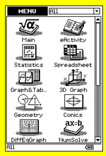
Write down some more pairs of twins that exhibit this behaviour.

4.1 EAT 7

Open the *ClassPad 300* sequence of geometry files `dsquara1 to dsquara4` (in the `dtsquare` folder) and run the animations that have been added.

How do these animations relate to the behaviour that you discussed previously?

Measure aspects of these constructions to confirm your suspicions.



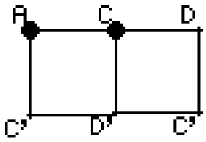
9.3

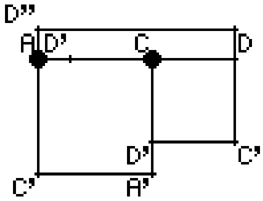
4.2 EAT 8

Repeat this for `dsquarb1` to `dsquarb4`.

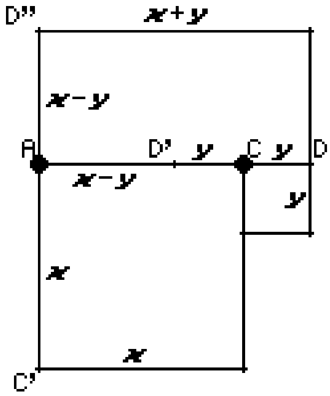
Write down an identity that summarises this behaviour.

Some snapshots of `dsquara4` are seen here, giving some indication of what is represented in this construction.



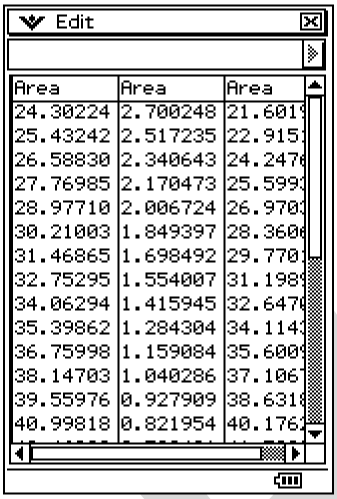


A labelled version of `dsquarb4` is shown below.



From the animation there seems to a link between the areas of the two squares and the area of the rectangle.

The square's area seems to be greater than the rectangle's area.



Area	Area	Area
24.30224	2.700248	21.601
25.43242	2.517235	22.915
26.58830	2.340643	24.247
27.76985	2.170473	25.599
28.97710	2.006724	26.970
30.21003	1.849397	28.360
31.46865	1.698492	29.770
32.75295	1.554007	31.198
34.06294	1.415945	32.647
35.39862	1.284304	34.114
36.75998	1.159084	35.600
38.14703	1.040286	37.106
39.55976	0.927909	38.631
40.99818	0.821954	40.176

To look more closely at these areas a table has been generated, showing the areas of the large square, the small square and the rectangle (in that order).

From it you can see that the area of the square minus the area of the square CD equals the area of the rectangle.

This result is hinted at in the snapshot at the top of this page, where the two squares are equal in size and the rectangle has 'disappeared'.

This identity is called the **Difference Of Two Squares**, for obvious reasons.

$$x^2 - y^2 = (x - y)(x + y)$$

Or to put it another way

$$(x - y)(x + y) = x^2 - y^2$$

4.3 Can you use your knowledge – 4?

1. Which of these are examples of the Difference of Two Squares identity?

1.	$x^2 - 36$	4.	$1 - k^2$	7.	$4x^2 - y^2$
2.	$(x-1)(x+1)$	5.	$(x-5)(x-5)$	8.	$16x^2 - 25y^4$
3.	$y^2 + 9$	6.	$(1-y)(y+1)$	9.	$-100x^2 - 81y^2$

2. Write an equivalent expression for the following

1.	$(x+4)(x-4)$	4.	$(2x+1)(2x-1)$	7.	$(-x+6)(-x-6)$
2.	$(y-7)(y+7)$	5.	$(y-3x)(y+3x)$	8.	$(xy+2z)(xy-2z)$
3.	$(2-x)(2+x)$	6.	$(5p-4q)(5p+4q)$	9.	$(8n+9p^2)(8n-9p^2)$

3. Write the following as the product of binomial factors.

1.	$x^2 - 25$	4.	$m^2 - n^2$	7.	$9x^2 - 36$
2.	$y^2 - 4$	5.	$4x^2 - y^2$	8.	$16a^2 - 49b^2$
3.	$9 - x^2$	6.	$a^2 - 144b^2$	9.	$81x^2 - 100y^2$

4. Write the following as the product of two factors.

1.	$x^2 - (x+3)$	4.	$(x+3)^2 - (x-2)^2$	7.	$(x+y)^2 - (x-y)^2$
2.	$(y-2)^2 - 4y^2$	5.	$(x-4)^2 - (x-6)^2$	8.	$(2x+y)^2 - (x-2y)^2$
3.	$x^2 - (1-x)^2$	6.	$(2x+3)^2 - (x+1)^2$	9.	$(m-2n)^2 - (n-m)^2$

By remembering that any number can be written as the square of its square root i.e.

$$a = \left(\sqrt{a}\right)^2$$

we can see the difference of two squares in many more places e.g.

$$x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5}) \quad \text{and} \quad 2y^2 - x = (\sqrt{2}y - \sqrt{x})(\sqrt{2}y + \sqrt{x})$$

5. Write the following in the form $(a-b)(a+b)$.

1.	$x^2 - 3$	4.	$m^2 - 2n^2$	7.	$7x^2 - 3w^4$
2.	$x^2 - 10$	5.	$5x^2 - 11y^2$	8.	$a^2b - c^2$
3.	$8 - y^2$	6.	$a^2 - b$	9.	$12x^2 - \left(\sqrt{3}x + 2\right)^2$

5. A mixed bag…

As a result of you work up to this point you should realise that, when faced with an algebraic expression, there are number of identities that can help you write down an equivalent expression. These identities include,

- The **Distributive Law** $a(b+c)=ab+ac$
  - Identification is aided by spotting a *common factor*.
- The **Perfect Square** identity $(a+b)^2=a^2+2ab+b^2$
- The **Difference of Two Squares** identity $a^2-b^2=(a-b)(a+b)$
- The binomial expansion process $(a+b)(c+d)=ac+ad+bc+bd$
  or for simple quadratic trinomials $(x+p)(x+q)=x^2+(p+q)x+pq$

At times the Distributive Law can be applied prior to the use of the other identities to aid in processes like 'factorisation', but this is not essential

$3x^2-24x+48$ $=3(x^2-8x+16)$ $=3(x-4)^2$	$3x^2-24x+48$ $=(3x-12)(x-4)$ $=3(x-4)(x-4)$ $=3(x-4)^2$

5.1 Can you use your knowledge – 5?

1. Write a 'brackets free' equivalent expression for each of the following.

1. $2(x+3y)$
2. $(y+3)(y-5)$
3. $-(x+4)^2$
4. $-x(2x-3)$
5. $-5(z-10)(z+10)$
6. $3(2x+1)(3x-5)$
7. $2y(y-6)(5-2y)$
8. $-3x(2x-9)^2$
9. $(w-4)^2+3(w+1)$
10. $4(2i+j)-i(3j-2)$
11. $(a^2b-3b)(a^2b+3b)$
12. $(x+2)^2-(x-3)^2$

3. Write these expressions as the product of factors

1. $5x^2-20x$
2. $-x^2-4x-4$
3. $z^2+4x+16$
4. $-3x^2+8x$
5. $16x^2+8xy+y^2$
6. $-x^2+100$
7. $8xy+12xy^2$
8. $75x^2-12y^2$
9. $-3t^2+27t-24$
10. $-2x^3+8xy^2$
11. $4a^2+24a+36$
12. $(x-1)^2-(1-3x)^2$

6. A case of mistaken identity?

6.1 EAT 9

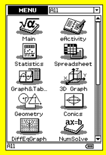
Consider the expression $\dfrac{2x^2 - x - 15}{x - 3}$

Study the value that this expression takes, for a wide range of x values.

Represent these values in a number of ways.

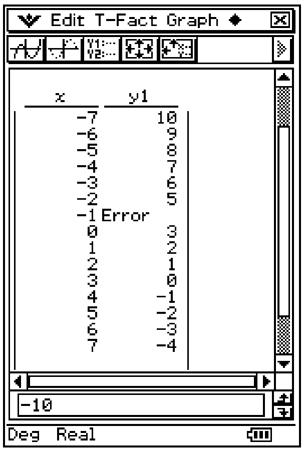
Suggest a *non-identical twin* for this expression.

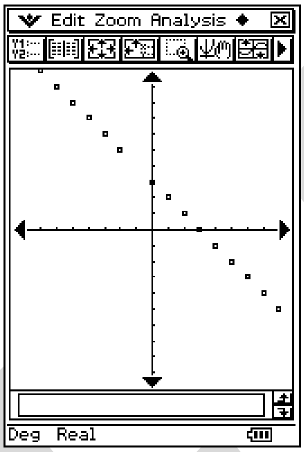
Present an argument supporting your suggestion.



9.4

Looking at expressions like $\dfrac{3+2x-x^2}{x+1}$ in a number of ways can be quite revealing.





As a table of values or as a graph, this complex looking expression behaves in a relatively simple way.

If you are familiar with linear or 'constant adder' patterns you might even suggest that, except for $x=-1$, this behaviour can be described by the expression $3-x$.

In effect this is making the conjecture that

$$\frac{3+2x-x^2}{x+1}=3-x \quad \text{for } x\neq -1$$

By writing down equivalent expressions, this conjecture can be proved i.e.

$$\begin{aligned}
&\frac{3+2x-x^2}{x+1}\\
&=\frac{-(x^2-2x-3)}{x+1}\\
&=\frac{-(x-3)(x+1)}{x+1}\\
&=-(x-3) \qquad \text{if} \quad x\neq -1\\
&=3-x
\end{aligned}$$

The third and fourth lines above are equivalent (i.e. return the same value for a given input) because, for example $\dfrac{-4\times 8}{8}=-4=-(7-3)$ (the case where $x=7$)

except in the case where $x=-1$ because $\dfrac{-(-4)\times 0}{0}$ cannot be defined but $-(-1-3)=4$.

Multiplying by zero generally gives the result of zero, regardless of other values.
Dividing by zero generally gives an infinitely large result, regardless of other values.
These conflicting processes mean that the expression cannot be defined for $x=-1$ so, in summary

$$\frac{3+2x-x^2}{x+1}=3-x \quad \text{for } x\neq -1$$

$$\frac{3+2x-x^2}{x+1} \quad \text{is } \textbf{undefined} \text{ for } x=-1$$

6.2 Can you use your knowledge – 5?

1. Use a similar approach to write down simpler looking non-identical twins for these expressions. Clearly identify the values for which these expressions are not equivalent to their simpler looking twin.

1.	$\dfrac{8x-4}{2x-1}$	5.	$\dfrac{x^2+5x}{x}$	9.	$\dfrac{x^2+5x+4}{x+1}$
2.	$\dfrac{10x+20}{2x+4}$	6.	$\dfrac{7x-2x^2}{x}$	10.	$\dfrac{x^2-7x+10}{x-2}$
3.	$\dfrac{24x+18}{8x-6}$	7.	$\dfrac{x^4-3x^2}{x^2}$	11.	$\dfrac{4x^2-12x+9}{2x-3}$
4.	$\dfrac{9y-15}{12y-20}$	8.	$\dfrac{x^2-1}{x-1}$	12.	$\dfrac{2x^2+3x-20}{x+4}$

For Example

$$\frac{16x^2-1}{4x^2-23x-6}$$

$$=\frac{(4x-1)(4x+1)}{(4x+1)(x-6)}$$

$$=\frac{4x-1}{x-6} \quad \textit{if} \;\; 4x+1\neq 0 \quad \textit{i.e.} \;\; x\neq -\frac{1}{4}$$

2. Tackle these in a similar way

1.	$\dfrac{x^2+5x-14}{x^2+7x}$	5.	$\dfrac{x^2+6x-16}{3x+24}$	9.	$\dfrac{9x^2-18x}{9x^2-36}$
2.	$\dfrac{x^2+2x-8}{x^2+5x+4}$	6.	$\dfrac{2x^2-7x+3}{x^2-6x+9}$	10.	$\dfrac{6x^2-19x+10}{4x^2-20x+25}$
3.	$\dfrac{x^2-4x+4}{x^2-4}$	7.	$\dfrac{2x^2+4x-30}{4x^2-12x}$	11.	$\dfrac{ax-ay}{bx^2-by^2}$
4.	$\dfrac{x^2+x}{1-x^2}$	8.	$\dfrac{x-1-x^2}{x^2-x+1}$	12.	$\dfrac{16x^2-y^2}{y^2-4xy}$

7. A funny looking twin indeed!

Asked for a non-identical mathematical twin for $x^2 + 6x + 8$ a student provided the answer $(x+3)^2 - 1$. Were they wrong?

When asked how this was obtained they replied

> "I knew the perfect square $(x+3)^2$ would give me $x^2 + 6x + \ldots$ (as required) but would also give $\ldots + 9$, and so I subtracted 1 to make it $\ldots + 8$."

This technique is called *Completing the Square,* as we create a perfect square $(x+a)^2$, and then complete it with a constant term, to construct an equivalent expression.

7.1 Can you use your knowledge – 6?

1. Which perfect square has a twin that looks like,

   1. $x^2 + 10x + \ldots$
   2. $x^2 - 10x + \ldots$
   3. $x^2 + 2x + \ldots$
   4. $y^2 + 18y + \ldots$
   5. $t^2 - 4t + \ldots$
   6. $k^2 - 30k + \ldots$
   7. $x^2 + 5x + \ldots$
   8. $x^2 - 3x + \ldots$
   9. $x^2 + 2ax + \ldots$

2. Complete the right hand side to make these expressions equivalent

   1. $x^2 + 8x + 7 = (x+4)^2 \ldots$
   2. $x^2 - 6x + 3 = (x-3)^2 \ldots$
   3. $x^2 + 14x - 9 = (x+7)^2 \ldots$
   4. $m^2 - 12m + 50 = (m-6)^2 \ldots$
   5. $y^2 - 20y = (y-10)^2 \ldots$
   6. $x^2 + x + 1 = (x + \frac{1}{2})^2 \ldots$
   7. $x^2 - 7x - 1 = (x-3.5)^2 \ldots$
   8. $x^2 - 0.8x - 0.2 = (x-0.4)^2 \ldots$

3. Use the method of *Completing the Square* to write down a non-identical mathematical twin of the form $(x+h)^2 + k$ for the following

   1. $x^2 + 4x + 1$
   2. $x^2 + 16x + 20$
   3. $x^2 - 10x - 10$
   4. $x^2 - 2x - 3$
   5. $x^2 + 5x + 2$
   6. $t^2 + 40t + 100$
   7. $x^2 + 9x + 9$
   8. $y^2 + 8y$
   9. $x^2 + 3x + 8$
   10. $x^2 - 7x - 10$
   11. $y^2 + y - 2$
   12. $x^2 - 11x - 5$

This technique can be extended so that is it applies to *all quadratic trinomials* by the careful use of the distributive law e.g.

$-x^2+3x-1$	$2x^2+8x-7$
$-(x^2-3x)-1$	$=2(x^2+8x)-7$
$-[(x-\frac{3}{2})^2-\frac{9}{4}]-1$	$=2[(x+4)^2-16]-7$
$-(x-\frac{3}{2})^2+\frac{9}{4}-1$	$=2(x+4)^2-32-7$
$-(x-\frac{3}{2})^2+\frac{5}{4}$	$=2(x+4)^2-39$

4. Use the method of *Completing the Square* to write down a non-identical mathematical twin of the form $a(x+h)^2+k$ for the following,

1.	$2x^2+16x+3$	5.	$-x^2+14x+10$	9.	$-3x^2+33x+8$
2.	$3x^2+30x-11$	6.	$-2x^2-24x+3$	10.	$5x^2-45x+20$
3.	$2x^2-8x$	7.	$-x^2-2x-5$	11.	$2x^2-3x+2$
4.	$5x^2-30x+2$	8.	$4x^2-12x+1$	12.	$2x^2+5x-3$

8. The magic of Algebra!

8.1 EAT 10

Think of any odd number.
Square it.
Write down the number one less than this square number.

Repeat for 4 other odd numbers.

What property do the 5 numbers that you have written down have in common?

Can you prove that the result of this process will always share this property?

Number tricks such as the one in EAT 10 can often be analysed using algebraic identities like those you have been studying.

You should have seen that if you

- Think of any odd number.
- Square it.
- Write down the number one less than this square number.

You seem always get a multiple of 4, or at least it seems that way.

The question is whether or not this result could be proved for the *infinite* number of odd numbers that we could think of.

If we *let* x represent any integer (some write this as $x \in R$).
Then the expression $2x+1$ represents *all odd integers*.
The square of this can now be written as a number of non-identical twins

$$(2x+1)^2$$
$$= 4x^2 + 4x + 1$$
$$= 4(x^2 + x) + 1$$

This last twin was carefully chosen to show that this square of any odd number is *always one greater than a multiple of 4!*

8.2 Can you use your knowledge – 5?

For these number tricks, do them three or four times so that you see what is going on, then prove it to be so for all cases…!

1. Choose any integer and write it down.
   Write down its square.
   Now write down the product of the integers "either side" your first number.

   What do you notice about the two numbers that you have written down?
   Can you prove it to be so for all cases?

2. Find the sum of any 5 consecutive integers and write it down.
   Repeat for three or four different sets of 5 consecutive integers.

   What property do these values share?
   Prove that is true for all possible sets of 5 consecutive integers.

3. Choose any integer.
   Multiply it by the integers "either side" of it.
   Add your original integer.
   Write down any ten consecutive integers that include this result.

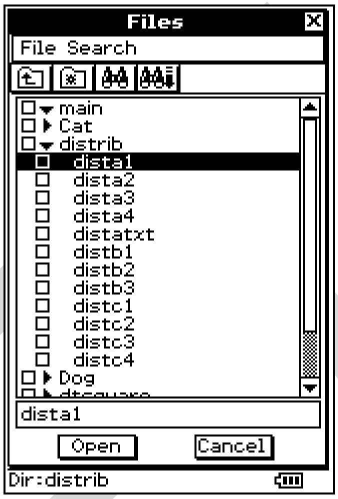
   Show you maths teacher and be amazed when they tell you what your original integer was!

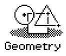
4. List the square numbers (starting with zero).
   Now list the gaps between consecutive square numbers.

   What do you notice?
   Prove it to be so.

9. eTech Support.

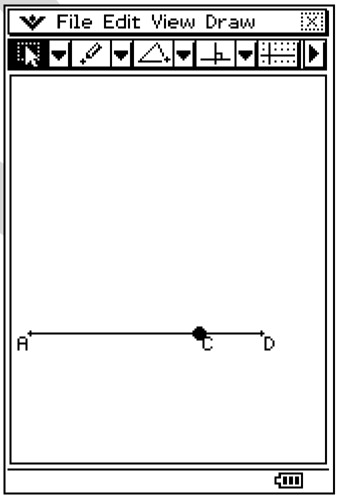
9.1 Opening a geometry file and animating it.



Enter the  mode of your *ClassPad 300*.
Go to the `File` drop down menu and tap `Open`.

View the contents of the folder that you require (on the right we see the contents of the `distrib` folder) by tapping on the black triangle (changing it to ▼).

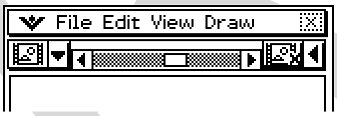
Tap on the file that you wish to open (i.e. `dista1`) and then click `Open`.



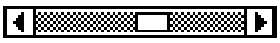
With a file that is able to be animated (like the ones in this unit), the sequence required to run an animation is,

- go to the `Edit` menu,
- select `Animate` and then tap
  - `Go (once)` – for a single "run through" or
  - `Go (to and fro)` – for continual animation (stopped by )

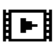
For more control over the animation, open the *Animation User Interface* by going to the View menu and tapping on `Animation UI`. This changes the tool bar to

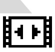


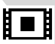
The standard toolbar is restored by repeating the above procedure

Using this interface the horizontal slider  can be used to see the animation 'frame by frame' – use the left and right arrows or drag the white box.

In addition the left drop down menu (tap ▼) can be used to control the animation with keys such as

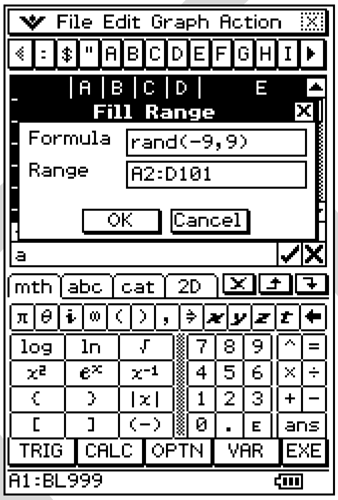
 `Go (once)`

 `Go (to and fro)`

 `Stop`

9.2 Tabulating an expression in a spreadsheet.

Enter the Spreadsheet mode of your ClassPad 300. Open a new spreadsheet by tapping `New` in the `File` menu. Type `a`, `b`, `c` and `d` into the first four cells in row `1`.
Type $(a+b)(c+d)$ into the fifth cell in row 1 (i.e. `E1`).
Type your non-identical mathematical twin in `F1`.



To fill the first 100 cells of columns with random integers between – 9 to 9 go to the `Edit` menu and choose the command **`Fill Range`**.

Enter the formula `rand(-9,9)` with a range of `A2:D101`.

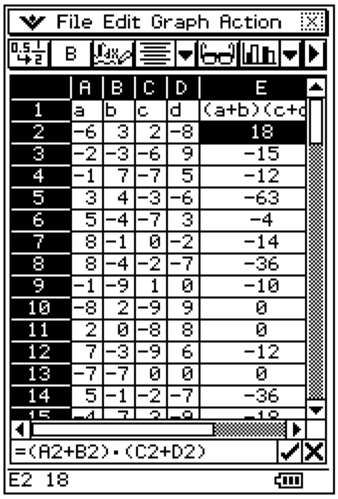
To compute the value of the expression $(a+b)(c+d)$ for the 100 sets of randomly chosen `a`, `b`, `c` and `d` values, enter this expression in Column E in the following way,

With the cursor in cell `E2` use the `fill range` command and enter the formula `=(A2+B2)×(C2+D2)` with a range of `E2` to `E101`.

Use a similar method to enter your non-identical mathematical twin in column `F` and then compare the two columns.

Another useful command to tidy up your spreadsheet is **`AutoFit Selection`** from the `Edit` menu.
With column selected, this command will modify the column width to fit its contents.

Cell alignment of selected cells can be changed from ▤ to ▤ using the drop down menu in the centre of the toolbar.

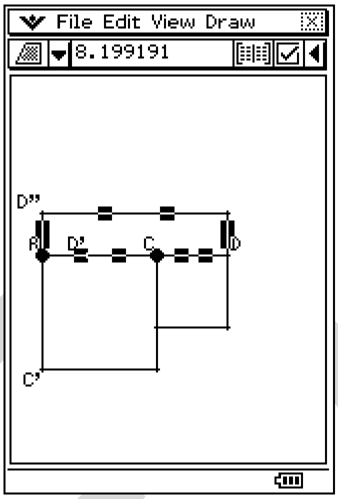


Your 'finished product' should look something like this.

If you wish to `Save` your spreadsheet you will find this command in the `File` menu.

9.3 Measuring aspects of geometric constructions.

Any aspect of a geometric construction can be measured by the *ClassPad 300*.



All you need to do is select the features that define the thing you want to measure.

For example, if you want to measure the area of a shape you need to select (by tapping) all the line segments that make it up, or alternatively select all its vertices.

With your selection made tap on the black triangle

on the right side of the tool bar [2].
This will take you to the measurement box as shown right.

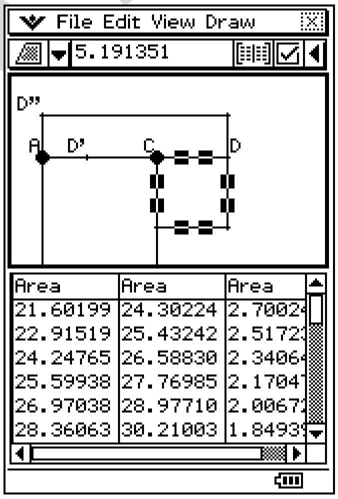
With the measurement box showing your selections can be changed and different aspects of the construction can be measured.

Note:

- Tap on blank space to deselect all.
- If the measurement box is blank you have not selected the features needed to define the aspect you wish to measure. Try again.

Tabulating measured quantities.

If you have previously run an animation that involved the thing you have measured, then the values of this (and every other) aspect of the construction is stored *for each 'step' of the animation*



With the measurement of interest visible in the measurement box tap on the table icon.

A table will be displayed in the newly split window.

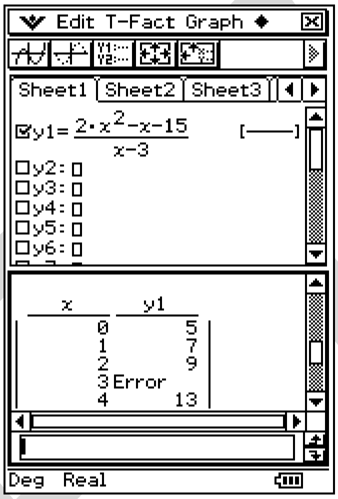
By repeating this process of selection, measurement and tabulation, the table can be added to, so that quantities can be compared.

With columns of the table selected, the `Edit` menu can be used to `copy` this data, which then can be `pasted` into a , amongst other places.

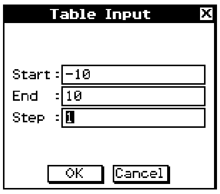
[2] The `Animation User Interface` (`UI`) may need to be turned off in the `View` menu

9.4 Tabulating an expression involving a single variable.

One of the easy ways to look at values that an algebraic expression takes for a range of inputs is by using the Graph&Tab mode of a *ClassPad 300*.



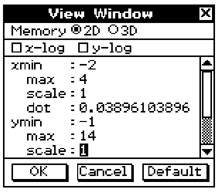
Enter the expression into `y1` as $(2x^2 - x - 15) \div (x - 3)$.



Tap on the Table Input button to set the `Table Input`.

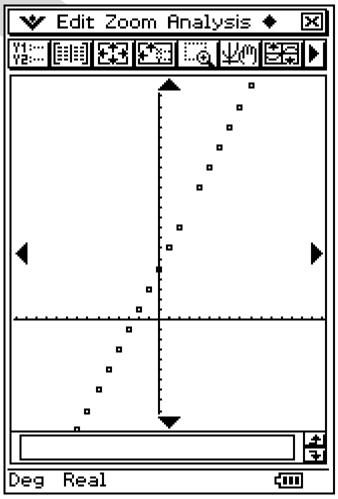
Tap on the Table button to see the table in the bottom of a newly split screen. Tap Resize if you wish to enlarge the table.

Plotting the values of an expression.



To set the `View Window` for a plot of the values in a table tap the View Window button.

Given the values seen in our table the settings seen left could be used in the drawing of a plot.



Tap the plot button to plot the values on your chosen View Window.

By first tapping Resize and then selecting `Square` from the `Zoom` menu a more comprehensive and accurate plot is obtained.

10. Answers.

1.4	**Can you use your knowledge – 1?**

2.1	$2(3x+1)$
2.2	$4(x+3)$ *or* $2(2x+6)$
2.3	$10(x+2)$ *or* $5(2x+4)$ *or* $2(5x+10)$
2.4	$3(x-4)$
2.5	$5(2x-25)$
2.6	$2(x+\frac{5}{2})$ *or* $4(\frac{1}{2}x+\frac{5}{4})$ *or* ...
2.7	$9(x+\frac{4}{9})$ *or* $4(\frac{9}{4}x+1)$ *or* ...
2.8	$-2(x+3)$ *or* $2(-x-3)$ *or* $-(2x+6)$
2.9	$-3(2x-3)$ *or* $3(-2x+3)$ *or* $-(6x-6)$
2.10	$8(4-x)$ *or* $4(8-2x)$ *or* $2(16-4x)$
2.11	$-5(1+3x)$ *or* $5(-1-3x)$ *or* $-(5+15x)$

3.1	$2x+2y$
3.2	$16x-24$
3.3	$xy+5x$
3.4	x^2+2x
3.5	$-4x+20$
3.6	$2x^2+4xy$
3.7	$-x^2+6x$
3.8	$5x^2y-5x^2$
3.9	$6x^3-9x^2y$
3.10	a^2b+2ab^2

3.11	$-45x^2y^3+5y^4$

4	$2m+2n+4$ $2(m+1)+2(n+1)$ $(m+2)(n+2)-mn$ $m+1+n+1+$ $+m+1+n+1$

5.1	$2x(x-4)$ $x(2x-8)$ $2(x^2-4x)$ x^2-8x+x^2 $8(x^2+x)-6x^2$
5.2	$8x^2+20xy$ $2(4x^2+10xy)$
5.3	$2(5-9x^2)$ $10(1-1.8x^2)$
5.4	$3z(y-8x)$ $z(3y-24x)$ $3(yz-8xz)$
5.5	$x^2(18-23y)$ $x(18x-23xy)$
5.6	$3x^2-3xy+9x$ $3(x^2-xy+3x)$
5.7	$2x^2+3xz-5x$ x^2+x^2+xy+ $+xy+xy-5x$
5.8	$4(3y+y^2)$ $4y(3+y)$ $y(12+4y)$
5.9	a^2b+ab^2 $a(ab+b^2)$ $b(a^2+ab)$
5.10	$x^2+7x-4x$ x^2+3x $x(x+7-4)$ $x(x+3)$
5.11	$10-x^2-3x$ $10-x^2-x-x-x$
5.12	$2x-10+xy+3x$ $x(y+5)-10$
5.13	$(x-4)(y+3)$ $x^2-4x+7x-28$ $x^2+3x-28$
5.14	$(1-z)(3+y)$ $3-3z+y-yz$
5.15	$zx-4z-8z+2xz$ $zx-12z+2xz$ $z(x-4)+2z(x-4)$ $3z(x-4)$ $3xz-12z$

2.3	**Can you use your knowledge – 2?**

2.1	$x^2+14x+49$
2.2	y^2+6y+9
2.3	x^2+2x+1
2.4	$z^2-20z+100$
2.5	$x^2-10x+25$
2.6	$4x^2+4x+1$
2.7	$y^2-12y+36$
2.8	$25x^2+40x+16$
2.9	$64-16w+w^2$
2.10	$25i^2+20ij+4j^2$
2.11	$16a^2-24ab+9b^2$
2.12	$36x^2-132xy+121$

3	1, 4, 6, 8, 9
4.1	$4x$

4.2	20
4.3	81
4.4	36
4.5	x^2
4.6	$24k$
4.7	$24xy$
4.8	$36x^2$
4.9	$14p^2q$

5.1	$x^2-10x+25$
5.2	$y^2+18y+81$
5.3	$a^2-2ab+b^2$
5.4	$9k^2+12km+4m^2$
5.5	$25p^2+50pq$ $+25q^2$
5.6	$16x^2-48xy^2$ $+36y^4$

6.1	Only one answer
6.2	$y^2+9y+\frac{81}{4}$
6.3	$a-2\sqrt{ab}+b^2$
6.4	$3k^2+4\sqrt{3}km+4m^2$
6.5	$25p^2+10p+1$
6.6	$16x^2-48xy+36y^2$

7.1	$(y+10)^2$
7.2	$(x-4)^2$
7.3	$(k-3)^2$
7.4	$(5+m)^2$
7.5	$(x+y)^2$
7.6	Not possible
7.7	Not possible
7.8	$(2a+6)^2$
7.9	$(3x-4y)^2$
7.10	$(5p+8q)^2$
7.11	$(x^2-9y)^2$
7.12	Not possible

3.2	**Can you use your knowledge – 3?**

2.1	$x^2+5x+2x+10$ $x^2+7x+10$
2.2	$y^2-6y-7y+42$ $y^2-13y+42$
2.3	$x^2-15x+x-15$ $x^2-14x-15$
2.4	$x^2+px+qx+pq$
2.5	$ab+3b-4a-12$
2.6	$2x^2-18x-3x+27$ $2x^2-21x+27$
2.7	$150x^2+15x$ $-10x-1$ $150x^2+5x-1$
2.8	$6y^2+8y+3y+4$ $6y^2+11y+4$
2.9	$15xy+12y$ $-10xz-8z$
2.10	$x^2-6x+6x-36$ x^2-36
2.11	$1+z-z-z^2$
2.12	$16a^2-12ab$ $+12ab-9b^2$ $16a^2-9b^2$

4.1	$(x+1)(x+7)$
4.2	$(x+8)(x+2)$
4.3	$(x+3)(x+4)$
4.4	$(x+5)(x-2)$
4.5	$(x-4)(x+1)$
4.6	$(x-4)(x-7)$
4.7	$(x-5)(x+4)$
4.8	$(x-3)(x-6)$
4.9	$(x+8)(x+3)$
4.10	$(x-4)(x-2)$
4.11	$(x-1)(x-9)$
4.12	$(x-12)(x+3)$

5.1	$(2x+1)(x+3)$
5.2	$(3x+5)(x+1)$
5.3	$(2x+3)(x+2)$
5.4	$(5x-2)(x-2)$
5.5	$(2x+1)(2x-5)$
5.6	$(3x-2)(2x+5)$
5.7	$(10x-7)(x+1)$
5.8	$(x-3)(6x-5)$
5.9	$(4x+1)(x+8)$
5.10	$(6x-7)(2x+3)$
5.11	$(5x+8)(2x-3)$
5.12	$(6x-7)(3x-2)$

4.3	**Can you use your knowledge – 4?**

1	1, 2, 4, 6, 7, 8
2.1	x^2-16
2.2	y^2-49
2.3	$4-x^2$
2.4	$4x^2-1$
2.5	y^2-9y^2
2.6	$25p^2-16q^2$
2.7	x^2-36
2.8	$x^2y^2-4z^2$
2.9	$64n^2-81p^4$

3.1	$(x-5)(x+5)$
3.2	$(y-2)(y+2)$
3.3	$(3-x)(3+x)$
3.4	$(m-n)(m+n)$
3.5	$(2x-y)(2x+y)$
3.6	$(a-12b)(a+12b)$
3.7	$(3x-6)(3x+6)$ $9(x-2)(x+2)$
3.8	$(4a-7b)(4a+7b)$
3.9	$(9x-10y)(9x+10y)$

4.1	$[x-(x+3)]$ $\times[x+(x+3)]$ $3(2x+3)$
4.2	$[(y-2)-2y]$ $\times[(y-2)+2y]$ $(-y-2)(3y-2)$ $-(y+2)(3y-2)$
4.3	$[x-(1-x)]$ $\times[x+(1-x)]$ $(2x+1)\times 1$
4.4	$[(x+3)-(x-2)]$ $\times[(x+3)+(x-2)]$ $5(2x+1)$
4.5	$[(x-4)-(x-6)]$ $\times[(x-4)+(x-6)]$ $2(2x+10)$ $4(x+5)$
4.6	$[(2x+3)-(x+1)]$ $\times[(2x+3)+(x+1)]$ $(x+2)(3x+4)$
4.7	$[(x+y)-(x-y)]$ $\times[(x+y)+(x-y)]$ $2y\times 2x$ $4xy$
4.8	$[(2x+y)-(x-2y)]$ $\times[(2x+y)+(x-2y)$ $(x+3y)(3x-y)$
4.9	$[(m-2n)-(n-m)]$ $\times[(m-2n)+(n-m$ $(2m-3n)(-n)$ $-n(2m-3n)$

5.1	$\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$
5.2	$\left(x-\sqrt{10}\right)\left(x+\sqrt{10}\right)$
5.3	$\left(\sqrt{8}-y\right)\left(\sqrt{8}+y\right)$ $\left(2\sqrt{2}-y\right)\left(2\sqrt{2}+y\right)$
5.4	$\left(m-\sqrt{2}n\right)\left(m+\sqrt{2}n\right)$
5.5	$\left(\sqrt{5}x-\sqrt{11}y\right)$ $\times\left(\sqrt{5}x+\sqrt{11}y\right)$
5.6	$\left(a-\sqrt{b}\right)\left(a+\sqrt{b}\right)$
5.7	$\left(\sqrt{7}x-\sqrt{3}w^2\right)$ $\times\left(\sqrt{7}x+\sqrt{3}w^2\right)$
5.8	$\left(a\sqrt{b}-c\right)\left(a\sqrt{b}+c\right)$
5.9	$\left[\sqrt{12}x-\left(\sqrt{3}x+2\right)\right]$ $\times\left[\sqrt{12}x+\left(\sqrt{3}+2\right)\right]$ $\left(2\sqrt{3}x-\sqrt{3}-2\right)$ $\times\left(2\sqrt{3}x+\sqrt{3}+2\right)$ $\left(\sqrt{3}x-2\right)\left(3\sqrt{3}x+2\right)$

5.1	**Can you use your knowledge – 5?**

1.1	$2x+6y$
1.2	$y^2-2y-15$
1.3	$-x^2-8x-16$
1.4	$-2x^2+3x$
1.5	$-5z^2+500$
1.6	$18x^2-21x-15$
1.7	$-4y^3+34y^2-60y$
1.8	$-12x^3+54x^2$ $-343x$
1.9	$w^2-5w+19$
1.10	$10i+4j-3ij$
1.11	$a^4b^2-9b^2$
1.12	$10x-5$

2.1	$5x(x-4)$
2.2	$-(x+2)^2$
2.3	$(z+2)(z+8)$
2.4	$-x(3x-8)$
2.5	$(4x+y)^2$
2.6	$-(x+10)(x-10)$
2.7	$4xy(2+3y)$
2.8	$3(5x-2y)$ $\times(5x+2y)$
2.9	$-3(t-1)(t-8)$
2.10	$-2x(x-2y)$ $\times(x+2y)$
2.11	$(2a+6)^2$
2.12	$-2x(4x-2)$

6.2	**Can you use your knowledge – 6 ?**

1.1	$\dfrac{4(2x-1)}{2x-1}$ $=4 \quad if \quad x\neq\frac{1}{2}$
1.2	$\dfrac{5(2x+4)}{2x+4}$ $=5 \quad if \quad x\neq -2$
1.3	$\dfrac{3(8x+6)}{8x+6}$ $=3 \quad if \quad x\neq -\frac{3}{4}$
1.4	$\dfrac{3(3y-5)}{4(3y-5)}$ $=\dfrac{3}{4} \quad if \quad y\neq\frac{5}{3}$
1.5	$\dfrac{x(x+5)}{x}$ $=x+5 \quad if \quad x\neq 0$
1.6	$=7-2x \quad if \quad x\neq 0$
1.7	$=x^2-3 \quad if \quad x\neq 0$
1.8	$\dfrac{(x+1)(x-1)}{x-1}$ $=x+1 \quad if \quad x\neq 1$

1.9	$\frac{(x+1)(x+4)}{x+1} = x+4$ *if* $x \neq -1$
1.10	$\frac{(x-2)(x-5)}{x-2} = x-5$ *if* $x \neq 2$
1.11	$\frac{(2x-3)^2}{2x-3} = 2x-3$ *if* $x \neq \frac{3}{2}$
1.12	$\frac{(2x-5)(x+4)}{x+4} = 2x-5$ *if* $x \neq -4$

2.1	$\frac{(x+7)(x-2)}{x(x+7)} = \frac{x-2}{x}$ *if* $x \neq -7$
2.2	$\frac{(x+4)(x-2)}{(x+4)(x+1)} = \frac{x-2}{x+1}$ *if* $x \neq -4$
2.3	$\frac{(x-2)^2}{(x-2)(x+2)} = \frac{x-2}{x+2}$ *if* $x \neq 2$
2.4	$\frac{x(x+1)}{(1-x)(1+x)} = \frac{x}{1-x}$ *if* $x \neq -1$
2.5	$\frac{(x+8)(x-2)}{3(x+8)} = \frac{x-2}{3}$ *if* $x \neq -8$
2.6	$\frac{(2x-1)(x-3)}{(x-3)^2} = \frac{2x-1}{x-3}$ *if* $x \neq 3$
2.7	$\frac{2(x-3)(x+5)}{4x(x-3)} = \frac{x+5}{2x}$ *if* $x \neq 3$
2.8	$\frac{-(x^2-x+1)}{x^2-x+1} = -1$ *as* $x^2-x+1 \neq 0$
2.9	$\frac{9x(x-2)}{9(x-2)(x+2)} = \frac{x}{x-2}$ *if* $x \neq 2$
2.10	$\frac{(2x-5)(3x-2)}{(2x-5)^2} = \frac{3x-2}{2x-5}$ *if* $x \neq \frac{5}{2}$
2.11	$\frac{a(x-y)}{b(x+y)(x-y)} = \frac{a}{b(x+y)}$ *if* $x \neq y$
2.12	$\frac{(4x-y)(4x+y)}{y(y-4x)} = \frac{-(4x+y)}{y}$ *if* $y \neq 4x$

7.1	**Can you use your knowledge – 7 ?**

1.1	$(x+5)^2$
1.2	$(x-5)^2$
1.3	$(x+1)^2$
1.4	$(y+9)^2$
1.5	$(t-2)^2$
1.6	$(k-15)^2$
1.7	$(x+\frac{5}{2})^2$
1.8	$(x-\frac{3}{2})^2$
1.9	$(x+a)^2$

2.1	-9
2.2	-6
2.3	-58
2.4	$+14$
2.5	-100
2.6	$+\frac{3}{4}$
2.7	$-\frac{53}{4}$
2.8	-0.36

3.1	$(x+2)^2+3$
3.2	$(x+8)^2-44$
3.3	$(x-5)^2-35$
3.4	$(x-1)^2+4$
3.5	$(x+\frac{5}{2})^2-\frac{17}{4}$
3.6	$(t+20)^2+300$
3.7	$(x+\frac{9}{2})^2-\frac{45}{4}$
3.8	$(y+4)^2-16$
3.9	$(x+\frac{3}{2})^2+\frac{23}{4}$
3.10	$(x-\frac{7}{2})^2-\frac{89}{4}$
3.11	$(y+\frac{1}{2})^2-\frac{9}{4}$
3.12	$(x-\frac{11}{2})^2-\frac{141}{4}$

4.1	$2(x+4)^2-29$
4.2	$3(x+5)^2-86$
4.3	$2(x-2)^2-8$
4.4	$5(x-3)^2+17$
4.5	$-(x-7)^2+59$
4.6	$-2(x+6)^2+75$
4.7	$-(x-1)^2-4$
4.8	$4(x-\frac{3}{2})^2-8$
4.9	$-3(x+\frac{11}{2})^2+\frac{395}{4}$
4.10	$5(x-\frac{9}{2})^2-\frac{325}{4}$
4.11	$2(x-\frac{3}{4})^2+\frac{7}{8}$
4.12	$2(x+\frac{5}{2})^2-\frac{49}{8}$