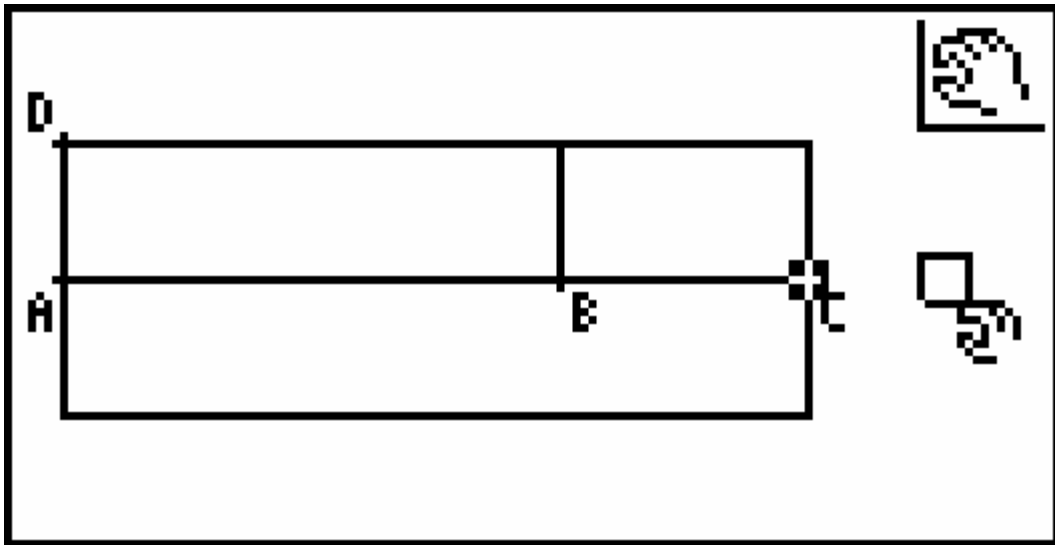


# Seeing Double

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# Seeing Double



Looking closely at algebraic identities.

Written by Alastair Lupton and Anthony Harradine.

# Seeing Double

Version 1.02 – April 2008.

Written by Anthony Harradine and Alastair Lupton.

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# Index

<i>Section</i>	<i>Page</i>
1. Look hard ... what do you see?.....	5
2. Looking at squares and other beasts. ....	8
3. And now speaking generally... ..	11
4. Another interesting case.....	13
5. A mixed bag.....	16
6. A case of mistaken identity? .....	17
7. A funny looking twin indeed! .....	20
8. The magic of Algebra!.....	22
9. eTech Support. ....	24
10. Answers.....	28

## Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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## Legend.

### EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just *explore and think!*

At certain points the learning process should have generated some **burning mathematical questions** that should be discussed and pondered, and then answered as you learn more!



### Time to Formalise.

These notes document the learning that has occurred to this point, using a degree of formal mathematical language and notation.



### Examples.

Illustrations of the mathematics at hand, used to answer questions.

# 1. Look hard ... what do you see?

Mathematical notation can be used to describe the behaviour of things that we see.

If used well it is a simple and powerful tool.

At best, behaviour can be fully captured by a few symbols – the fewer the better!

## 1.1 EAT 1

Open the CASIO 9860G AU geometry file `DISTA1`. The file `DISTGEOM` may need to be copied from the `10ALG1ID` folder in your calculator's storage memory.

Run the animation that has been added.

Your task is to describe / 'name' the thing(s) you see.

It may help you to think about **what changes** and **what stays the same**.

**Record your descriptions.**

**Compare and discuss these with your class.**



9.1

## 1.2 EAT 2

Repeat this for `DISTA2` and `DISTA3`.

**Record and discuss the *different* ways that what you see can be described.**

## 1.3 EAT 3

Explore and think in the same way about the sequence of animated geometry files `DISTB1`, `DISTB2` and `DISTB3`.

Explore and think in the same way about the sequence of animated geometry files `DISTC1`, `DISTC2` and `DISTC3`.

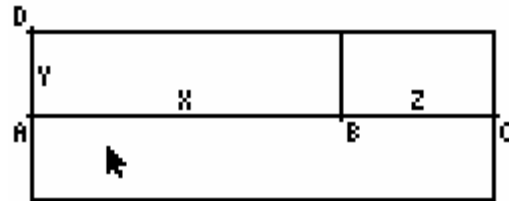
One group who looked at the previous animations labelled the diagram (below) and saw

$$xy + yz$$

$$y \times (x + z)$$

Can you see what they are describing?

Can you see how they chose to describe it?



Another group saw

$$2y \times (x + z)$$

$$2(xy + yz)$$

$$xy + yz + xy + yz$$

$$2xy + 2yz$$

$$xy + yz + y \times (x + z)$$



Can you see what they were describing?

Can you see how they chose to describe it?

These algebraic expressions can be thought of as *non-identical mathematical twins*. They don't *look* the same but they are *essentially* the same, equivalent in value and, in a way, interchangeable<sup>1</sup>

### Another way to look at this ...

The 'equivalence' of these twins means that they take the same value when values for the variables are assigned. This assigning of values can be shown via spreadsheet.

Open the 9860 spreadsheet file `DISTRIB`. The file `DISTSSHT` may need to be copied from the `10ALG`1ID` folder in your calculator's Storage Memory.

DIST	A	B	C	D	E
1	a	b	c	a(b+c)	ab+ac
2	-2	-6	9	21	21
3	-5	-2	7	-10	-10
4	2	1	7	16	16
5	-1	-9	1	56	56

`=A5*B5+A5*C5`

This spreadsheet focuses on the twins described by the first group (above). Change values for A, B and C to see equivalence in action.

If we are sure that these expressions return the same value for **all** possible values that the variables could take (a spreadsheet with infinite rows perhaps?) then we can say that they are *equivalent* and they can be expressed as an equality.

---

<sup>1</sup> Unlike human twins, who are unique individuals in their own right and are in no way interchangeable!

$$a(b + c) = ab + ac$$

or to put it another way,

$$ab + ac = a(b + c)$$

This is commonly called the **Distributive Law**



Note: Our demonstrations do not prove the equivalence needed to make this a mathematical law, but others have provided the necessary proof.

(One such proof can be found at [http://academic.gallaudet.edu/courses/MAT/MAT000Ivew.nsf/ID/918f9bc4dda7eb1c8525688700561c74/\\$file/NUMBERS.pdf](http://academic.gallaudet.edu/courses/MAT/MAT000Ivew.nsf/ID/918f9bc4dda7eb1c8525688700561c74/$file/NUMBERS.pdf))

#### 1.4 Can you use your knowledge – 1?

1. Write down 6 sets of non-identical mathematical twins that illustrate the Distributive Law.

2. Write each of the following as a product of two factors

1.  $6x + 2$

5.  $10x - 25$

9.  $-6x + 9$

2.  $4x + 12$

6.  $2x + 5$

10.  $32 - 8x$

3.  $10x + 20$

7.  $9x - 4$

11.  $-5 - 15x$

4.  $9x - 12$

8.  $-2x - 6$

3. Write each of the following as the sum of two elements

1.  $2(x + y)$

5.  $-4(x - 5)$

9.  $3x^2(2x - 3y)$

2.  $8(2x - 3)$

6.  $2x(x + 2y)$

10.  $ab(a + 2b)$

3.  $x(y + 5)$

7.  $-x(x - 6)$

11.  $-5y^3(9x^2 - y)$

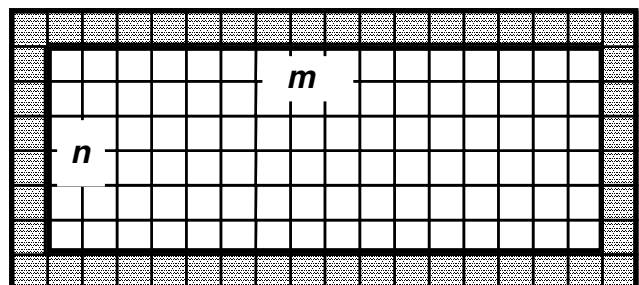
4.  $x(x + 2)$

8.  $5x^2(y - 1)$

4. Picture this – a nicely tiled, rectangular swimming pool with a tiled border which is one tile wide.

If the pool is  $m$  tiles long and  $n$  tiles wide. How many border tiles are there?

Derive as many expressions for the number of border tiles as you can



5. Write down one or more non-identical mathematical twins for each of the following expressions

- |                     |                     |                            |
|---------------------|---------------------|----------------------------|
| 1. $2x^2 - 8x$      | 6. $3x(x - y + 3)$  | 11. $10 - x(x - 3)$        |
| 2. $4x(2x + 5y)$    | 7. $x(2x + 3z - 5)$ | 12. $2(x - 5) + x(y + 3)$  |
| 3. $10 - 18x^2$     | 8. $12y + 4y^2$     | 13. $x(x - 4) + 7(x - 4)$  |
| 4. $3yz - 24xz$     | 9. $ab(a + b)$      | 14. $3(1 - z) + y(1 - z)$  |
| 5. $18x^2 - 23x^2y$ | 10. $x(x + 7) - 4x$ | 15. $z(x - 4) - 2z(4 - x)$ |

6. **Research** the mathematical terms *expand* and *factorise*.

Explain their meanings in your own words (including examples as required).

## 2. Looking at squares and other beasts.

### 2.1 EAT 4

Open the sequence of 9860 geometry files PSQUARA1 to PSQUARA4.

The file PSQUGEOM may need to be copied from the 10ALG'ID folder in your calculator's storage memory. Run the animations that have been added.

As before, your task is to describe / 'name' the thing(s) you see.

**Record your descriptions.**

**Compare and discuss these with your class.**



9.1

### 2.2 EAT 5

Repeat this for PSQUARB1 to PSQUARB4.

Summarise your findings from these activities.

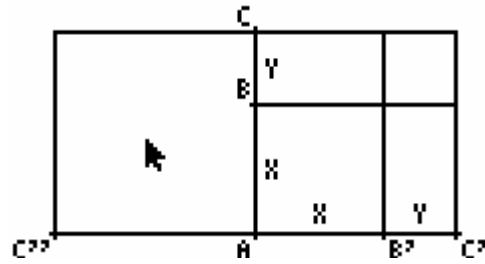


A 'snapshot' of PSQUARBY is shown here, with some labels added.

One group saw

$$x^2 + xy + xy + y^2 \quad \text{or}$$

$$x^2 + 2xy + y^2$$



**Can you see what they are describing?**  
**Can you see how they chose to describe it?**

Another group saw

$$(x + y)^2$$

**Can you see what they are describing?**  
**Can you see how they chose to describe it?**

This implies that

$$(x + y)^2 = x^2 + 2xy + y^2$$

or to put it another way,

$$x^2 + 2xy + y^2 = (x + y)^2$$

This is known as the **Perfect Square Identity**.



**But why doesn't**  $(x + y)^2 = x^2 + y^2$ ?

This common misunderstanding seems to follow from the correct application of the Distributive Law that says  $2(x + y) = 2x + 2y$ .

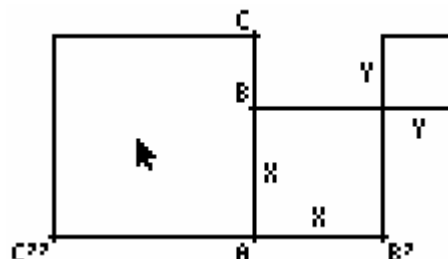
As it turns out, however, you can't "distribute" an operation like "squared" in the same way that you distribute a multiplication like  $2(\dots)$

If  $(x + y)^2 = x^2 + y^2$  then the large square would have the same area as the two smaller squares combined (see right).

*Clearly, that is not the case!*

There are two rectangles 'missing'.

Each has an area of  $xy$ , so there is  $2xy$  'missing'.



Another way to see this is in a spreadsheet. Open the spreadsheet file PSQUARE. You may need to copy the PSQUSSHT file from the DIALOG'D folder in your calculator's storage memory. This spreadsheet looks like,

PSQ	C	D	E	F
1	$(a+b)^2$	$a^2+b^2$	$2ab$	$a^2+2a$
2	4	34	-30	4
3	121	65	56	121
4	25	53	-28	25
5	1	13	-12	1
=A5^2+2*A5*B5+B5^2				
FILE EDIT DEL INS CLR D				

You can see that  $(a + b)^2$  (column C) is not equal to  $a^2 + b^2$  (column D) and that what is 'missing' is  $2ab$  (column E).

### 2.3 Can you use your knowledge – 2?

1. Write down 6 examples of the *Perfect Square Identity* – include a minus sign in at least 2 of them.

2. Write each of the following as the sum of three terms.

- |                 |                 |                    |
|-----------------|-----------------|--------------------|
| 1. $(x + 7)^2$  | 5. $(x - 5)^2$  | 9. $(8 - w)^2$     |
| 2. $(y + 3)^2$  | 6. $(2x + 1)^2$ | 10. $(5i + 2j)^2$  |
| 3. $(x + 1)^2$  | 7. $(y - 6)^2$  | 11. $(4a - 3b)^2$  |
| 4. $(z - 10)^2$ | 8. $(5x + 4)^2$ | 12. $(6x - 11y)^2$ |

3. Which of the following are examples of the *Perfect Square Identity*?

- |                     |                      |                           |
|---------------------|----------------------|---------------------------|
| 1. $x^2 - 14x + 49$ | 4. $(72x - 13y)^2$   | 7. $x^2 - 12x - 36$       |
| 2. $y^2 + 9$        | 5. $y^2 - 25x + 10$  | 8. $a^2 + 16b + 64$       |
| 3. $z^2 + 4z + 16$  | 6. $121 - 22z + z^2$ | 9. $9i^2 + 12ij^2 + 4j^4$ |

4. Add the missing term to make the following *Perfect Squares*.

- |                                    |                                    |                                       |
|------------------------------------|------------------------------------|---------------------------------------|
| 1. $x^2 + \underline{\quad} + 4$   | 4. $a^2 - 12a + \underline{\quad}$ | 7. $16x^2 + \underline{\quad} + 9y^2$ |
| 2. $x^2 - \underline{\quad} + 100$ | 5. $\underline{\quad} - 10x + 25$  | 8. $25w^2 - 60wx + \underline{\quad}$ |
| 3. $y^2 + 18y + \underline{\quad}$ | 6. $4k^2 + \underline{\quad} + 36$ | 9. $p^4 + \underline{\quad} + 49q^2$  |

5. Change a term to make these *Perfect Squares*

- |                     |                         |                           |
|---------------------|-------------------------|---------------------------|
| 1. $x^2 - 10x - 25$ | 3. $a - 2ab + b^2$      | 5. $25p^2 + 50pq + 1$     |
| 2. $y^2 + 9y + 81$  | 4. $3k^2 + 12km + 4m^2$ | 6. $16x^2 - 48xy + 36y^4$ |

6. For which parts of question 5 is there more than one correct answer?  
For those parts, provide an additional answer.
7. Where possible, write the following in the form  $(a + b)^2$
- |                      |                      |                            |
|----------------------|----------------------|----------------------------|
| 1. $y^2 + 20y + 100$ | 5. $x^2 + 2xy + y^2$ | 9. $9x^2 - 24xy + 16y^2$   |
| 2. $x^2 - 8x + 16$   | 6. $a^2 - 12a - 144$ | 10. $25p^2 + 80pq + 64q^2$ |
| 3. $k^2 - 6k + 9$    | 7. $1 - 2z - z^2$    | 11. $x^4 - 18x^2y + 81y^2$ |
| 4. $25 + 10m + m^2$  | 8. $4a^2 + 24a + 36$ | 12. $49a^2 - 42ab + 36b^2$ |

### 3. And now speaking generally...

$(a + b)^2$ , and its expansion  $a^2 + 2ab + b^2$ , is an example of a process sometimes called *binomial expansion*.

Roughly translated *binomial* means *two number*. The phrase *binomial expansion* is used in situations where

*a factor with two terms multiplied by another factor with two terms.*

What we have learned so far covered only the case where the two factors are identical (hence the 'squared'). What about binomial expansion in general?

#### 3.1 EAT 6

Consider the general binomial expansion  $(a + b)(c + d)$ .

What sort of construction or shape would be associated with this expression?

Can you draw it?

What does this suggest about a *non-identical twin expression* for  $(a + b)(c + d)$ ?

Check that any non-identical twin does in fact return the same value as

$(a + b)(c + d)$  for range of different inputs. A spreadsheet might help.



#### 9.2

Can you prove that these expressions are equivalent, using only the identities studied so far in the topic?

### 3.2 Can you use your knowledge – 3?

1. Write down 6 sets of non-identical mathematical twins where one is of the form  $(a + b)(c + d)$ .

2. Write each of the following as the sum of four terms, and then collect like terms where possible.

- |                      |                         |                          |
|----------------------|-------------------------|--------------------------|
| 1. $(x + 5)(x + 2)$  | 5. $(a + 3)(b - 4)$     | 9. $(5x + 4)(3y - 2z)$   |
| 2. $(y - 6)(y - 7)$  | 6. $(2x - 3)(x - 9)$    | 10. $(x - 6)(x + 6)$     |
| 3. $(x + 1)(x - 15)$ | 7. $(15x - 1)(10x + 1)$ | 11. $(1 + z)(1 - z)$     |
| 4. $(x + p)(x + q)$  | 8. $(3y + 4)(2y + 1)$   | 12. $(4a - 3b)(4a + 3b)$ |

3. Look at Question 2 parts 1 to 4 (above).

These expressions with three terms are called *quadratic trinomials* as they contain *three terms*, a quadratic term containing  $x^2$ , a linear term containing  $x$  and a constant term (independent of  $x$ ).

- Explain how the coefficient of linear term can be obtained.
- Explain how the constant term can be obtained.

4. Use this "sum and product" property to write the following as the product of two binomial factors.

- |                     |                     |                     |
|---------------------|---------------------|---------------------|
| 1. $x^2 + 8x + 7$   | 5. $x^2 - 3x - 4$   | 9. $x^2 + 11x + 24$ |
| 2. $x^2 + 10x + 16$ | 6. $x^2 - 11x + 28$ | 10. $x^2 - 6x + 8$  |
| 3. $x^2 + 7x + 12$  | 7. $x^2 - x - 20$   | 11. $x^2 - 10x + 9$ |
| 4. $x^2 + 3x - 10$  | 8. $x^2 - 9x + 18$  | 12. $x^2 - 9x - 36$ |

5. In a similar way, write the following as the product of two binomial factors.

- |                     |                      |                        |
|---------------------|----------------------|------------------------|
| 1. $2x^2 + 7x + 3$  | 5. $4x^2 - 8x - 5$   | 9. $4x^2 + 33x + 8$    |
| 2. $3x^2 + 8x + 5$  | 6. $6x^2 + 11x - 10$ | 10. $12x^2 + 4x - 21$  |
| 3. $2x^2 + 7x + 6$  | 7. $10x^2 + 3x - 7$  | 11. $10x^2 + x - 24$   |
| 4. $5x^2 - 12x + 4$ | 8. $6x^2 - 23x + 15$ | 12. $18x^2 - 33x + 14$ |

This process is sometimes called **binomial expansion** and is summarised by the identity

$$(a + b)(c + d) = ac + ad + bc + bd$$

When dealing with quadratic trinomials it is sometimes written as

$$(x + p)(x + q) = x^2 + (p + q)x + pq$$



## 4. Another interesting case...



Look closely at Question 2 parts 10, 11 and 12 on the previous page. You may notice that their non-identical twins differ from the twins in the other parts of this question.

Discuss with your classmates the way that these twins differ from the others that you have seen.

Discuss what causes this difference to come about.



Write down some more pairs of twins that exhibit this behaviour.

### 4.1 EAT 7

Open the sequence of 9860 geometry files `DSQUARA1` to `DSQUARA4`. The file `DSQUGEDM` may need to be copied from the `10ALG'ID` folder in your calculator's storage memory. Run the animations that have been added.

**How do these animations relate to the behaviour that you discussed previously?**

**Measure aspects of these constructions to confirm your suspicions.**



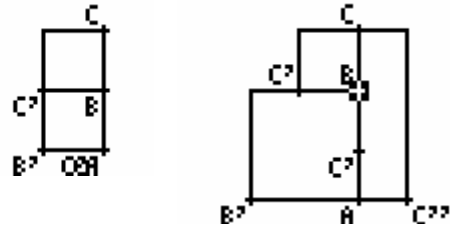
9.3

### 4.2 EAT 8

Repeat this for `DSQUARB1` to `DSQUARB4`.

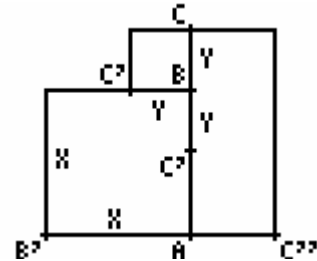
Write down an identity that summarises this behaviour.

Some snapshots of DSQUARA4 are seen here, giving some indication of what is represented in this construction.



A labelled version of DSQUARB4 is shown (right).

From the animation there seems to be a link between the areas of the two squares and the area of the rectangle. You can probably see that the area of the two squares combined is greater than the rectangle's area. Can you see a more exact relationship?



Area	Area	Area
7.0364	3.0166	4.0198
8.0775	3.2022	4.8753
9.1904	3.3933	5.7971
10.375	3.59	6.7851
11.631	3.7922	7.8394
12.96	4	8.96

To look more closely at these areas a table has been generated from the animation of DSQUARB4. It shows the areas of the 'top' square, the 'bottom' square and the rectangle (in that order).

From it you can see that the area of the 'top' square minus the area of the 'bottom' square equals the area of the rectangle.

This result is hinted at in the snapshot at the top of this page, where the two squares are equal in size and the rectangle has 'disappeared'.

This identity is called the **Difference Of Two Squares**, for obvious reasons.

$$x^2 - y^2 = (x - y)(x + y)$$

Or to put it another way

$$(x - y)(x + y) = x^2 - y^2$$



### 4.3 Can you use your knowledge – 4?

1. Which of these are examples of the *Difference of Two Squares* identity?

- |                     |                     |                      |
|---------------------|---------------------|----------------------|
| 1. $x^2 - 36$       | 4. $1 - k^2$        | 7. $4x^2 - y^2$      |
| 2. $(x - 1)(x + 1)$ | 5. $(x - 5)(x - 5)$ | 8. $16x^2 - 25y^4$   |
| 3. $y^2 + 9$        | 6. $(1 - y)(y + 1)$ | 9. $-100x^2 - 81y^2$ |

2. Write an equivalent expression for the following

- |                     |                         |                             |
|---------------------|-------------------------|-----------------------------|
| 1. $(x + 4)(x - 4)$ | 4. $(2x + 1)(2x - 1)$   | 7. $(-x + 6)(-x - 6)$       |
| 2. $(y - 7)(y + 7)$ | 5. $(y - 3x)(y + 3x)$   | 8. $(xy + 2z)(xy - 2z)$     |
| 3. $(2 - x)(2 + x)$ | 6. $(5p - 4q)(5p + 4q)$ | 9. $(8n + 9p^2)(8n - 9p^2)$ |

3. Write the following as the product of binomial factors.

- |               |                   |                     |
|---------------|-------------------|---------------------|
| 1. $x^2 - 25$ | 4. $m^2 - n^2$    | 7. $9x^2 - 36$      |
| 2. $y^2 - 4$  | 5. $4x^2 - y^2$   | 8. $16a^2 - 49b^2$  |
| 3. $9 - x^2$  | 6. $a^2 - 144b^2$ | 9. $81x^2 - 100y^2$ |

4. Write the following as the product of two factors.

- |                       |                             |                              |
|-----------------------|-----------------------------|------------------------------|
| 1. $x^2 - (x + 3)^2$  | 4. $(x + 3)^2 - (x - 2)^2$  | 7. $(x + y)^2 - (x - y)^2$   |
| 2. $(y - 2)^2 - 4y^2$ | 5. $(x - 4)^2 - (x - 6)^2$  | 8. $(2x + y)^2 - (x - 2y)^2$ |
| 3. $x^2 - (1 - x)^2$  | 6. $(2x + 3)^2 - (x + 1)^2$ | 9. $(m - 2n)^2 - (n - m)^2$  |

By remembering that any number can be written as the square of its square root i.e.

$$a = (\sqrt{a})^2$$

we can see the difference of two squares in many more places e.g.

$$x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5}) \quad \text{and} \quad 2y^2 - x = (\sqrt{2}y - \sqrt{x})(\sqrt{2}y + \sqrt{x})$$

5. Write the following in the form  $(a - b)(a + b)$ .

- |               |                   |                                |
|---------------|-------------------|--------------------------------|
| 1. $x^2 - 3$  | 4. $m^2 - 2n^2$   | 7. $7x^2 - 3w^4$               |
| 2. $x^2 - 10$ | 5. $5x^2 - 11y^2$ | 8. $a^2b - c^2$                |
| 3. $8 - y^2$  | 6. $a^2 - b$      | 9. $12x^2 - (\sqrt{3}x + 2)^2$ |

## 5. A mixed bag...

As a result of your work up to this point you should realise that, when faced with an algebraic expression, there are number of identities that can help you write down an equivalent expression. These identities include,

- The **Distributive Law**  $a(b + c) = ab + ac$ 
  - Identification of the expanded form is aided by spotting a *common factor*.
- The **Perfect Square** identity  $(a + b)^2 = a^2 + 2ab + b^2$
- The **Difference of Two Squares** identity  $a^2 - b^2 = (a - b)(a + b)$
- The binomial expansion process  $(a + b)(c + d) = ac + ad + bc + bd$   
or for simple quadratic trinomials  $(x + p)(x + q) = x^2 + (p + q)x + pq$

At times the Distributive Law can be applied prior to the use of the other identities to aid in processes like 'factorisation', but this is not essential

$$\begin{aligned}3x^2 - 24x + 48 \\ &= 3(x^2 - 8x + 16) \\ &= 3(x - 4)^2\end{aligned}$$

$$\begin{aligned}3x^2 - 24x + 48 \\ &= (3x - 12)(x - 4) \\ &= 3(x - 4)(x - 4) \\ &= 3(x - 4)^2\end{aligned}$$

### 5.1 Can you use your knowledge – 5?

1. Write a 'brackets free' equivalent expression for each of the following.

- |                     |                         |                              |
|---------------------|-------------------------|------------------------------|
| 1. $2(x + 3y)$      | 5. $-5(z - 10)(z + 10)$ | 9. $(w - 4)^2 + 3(w + 1)$    |
| 2. $(y + 3)(y - 5)$ | 6. $3(2x + 1)(3x - 5)$  | 10. $4(2i + j) - i(3j - 2)$  |
| 3. $-(x + 4)^2$     | 7. $2y(y - 6)(5 - 2y)$  | 11. $(a^2b - 3b)(a^2b + 3b)$ |
| 4. $-x(2x - 3)$     | 8. $-3x(2x - 9)^2$      | 12. $(x + 2)^2 - (x - 3)^2$  |

2. Write these expressions as the product of factors

- |                     |                        |                              |
|---------------------|------------------------|------------------------------|
| 1. $5x^2 - 20x$     | 5. $16x^2 + 8xy + y^2$ | 9. $-3t^2 + 27t - 24$        |
| 2. $-x^2 - 4x - 4$  | 6. $-x^2 + 100$        | 10. $-2x^3 + 8xy^2$          |
| 3. $z^2 + 10z + 16$ | 7. $8xy + 12xy^2$      | 11. $4a^2 + 24a + 36$        |
| 4. $-3x^2 + 8x$     | 8. $75x^2 - 12y^2$     | 12. $(x - 1)^2 - (1 - 3x)^2$ |



## 6. A case of mistaken identity?

### 6.1 EAT 9

Consider the expression  $\frac{2x^2 - x - 15}{x - 3}$

Study the values that this expression takes, for a wide range of  $x$  values.

Represent these values in a number of ways.

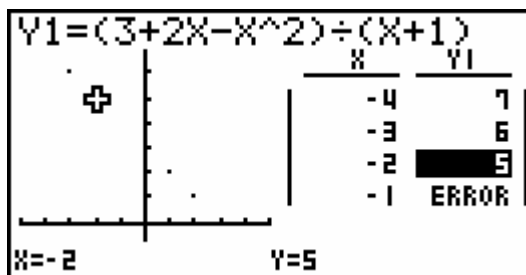
Suggest a *non-identical twin* for this expression.

Present an argument supporting your suggestion.



9.4

Looking at expressions like  $\frac{3+2x-x^2}{x+1}$  in a number of ways can be quite revealing.



As a table of values or as a graph, this complex looking expression behaves in a relatively simple way.

If you are familiar with linear or 'constant adder' patterns you might even suggest that, except for  $x = -1$ , this behaviour can be described by the expression  $3 - x$ .

In effect this is making the conjecture that

$$\frac{3+2x-x^2}{x+1} = 3-x \quad \text{for } x \neq -1$$

By writing down equivalent expressions, this conjecture can be proved i.e.

$$\begin{aligned} & \frac{3+2x-x^2}{x+1} \\ &= \frac{-(x^2-2x-3)}{x+1} \\ &= \frac{-(x-3)(x+1)}{x+1} \\ &= -(x-3) \quad \text{if } x \neq -1 \\ &= 3-x \quad \text{if } x \neq -1 \end{aligned}$$

The third and fourth lines above are equivalent for *almost all* values of  $x$ .

They return the same value for given input,

e.g.  $\frac{-4 \times 8}{8} = -4$  and  $-(7-3) = -4$  (the case where  $x = 7$ ).

This is not so *only in the case where*  $x = -1$ , because  $\frac{-(-4) \times 0}{0}$  cannot be defined

but  $-(-1-3) = 4$ .

Multiplying by zero generally gives the result of zero, regardless of other values.

Dividing by zero generally gives an infinitely large result, regardless of other values.

These conflicting processes mean that this expression cannot be defined for  $x = -1$ .

So, in summary,

$$\begin{aligned} & \frac{3+2x-x^2}{x+1} = 3-x \quad \text{for } x \neq -1 \\ & \frac{3+2x-x^2}{x+1} \text{ is } \mathbf{undefined} \text{ for } x = -1 \end{aligned}$$

## 6.2 Can you use your knowledge – 5?

1. Use a similar approach to write down simpler looking non-identical twins for these expressions. Clearly identify the values for which these expressions are not equivalent to their simpler looking twin.

1.  $\frac{8x-4}{2x-1}$

5.  $\frac{x^2+5x}{x}$

9.  $\frac{x^2+5x+4}{x+1}$

2.  $\frac{10x+20}{2x+4}$

6.  $\frac{7x-2x^2}{x}$

10.  $\frac{x^2-7x+10}{x-2}$

3.  $\frac{24x+18}{8x-6}$

7.  $\frac{x^4-3x^2}{x^2}$

11.  $\frac{4x^2-12x+9}{2x-3}$

4.  $\frac{9y-15}{12y-20}$

8.  $\frac{x^2-1}{x-1}$

12.  $\frac{2x^2+3x-20}{x+4}$

### For Example

$$\begin{aligned} & \frac{16x^2-1}{4x^2-23x-6} \\ &= \frac{(4x-1)(4x+1)}{(4x+1)(x-6)} \\ &= \frac{4x-1}{x-6} \quad \text{if } 4x+1 \neq 0 \quad \text{i.e. } x \neq -\frac{1}{4} \end{aligned}$$

2. Tackle these in a similar way.

1.  $\frac{x^2+5x-14}{x^2+7x}$

5.  $\frac{x^2+6x-16}{3x+24}$

9.  $\frac{9x^2-18x}{9x^2-36}$

2.  $\frac{x^2+2x-8}{x^2+5x+4}$

6.  $\frac{2x^2-7x+3}{x^2-6x+9}$

10.  $\frac{6x^2-19x+10}{4x^2-20x+25}$

3.  $\frac{x^2-4x+4}{x^2-4}$

7.  $\frac{2x^2+4x-30}{4x^2-12x}$

11.  $\frac{ax-ay}{bx^2-by^2}$

4.  $\frac{x^2+x}{1-x^2}$

8.  $\frac{x-1-x^2}{x^2-x+1}$

12.  $\frac{16x^2-y^2}{y^2-4xy}$

## 7. A funny looking twin indeed!

Asked for a non-identical mathematical twin for  $x^2 + 6x + 8$ , a student provided the answer  $(x + 3)^2 - 1$ . Were they wrong?

When asked how this was obtained they replied

"I knew the perfect square  $(x + 3)^2$  would give me  $x^2 + 6x + \dots$  (as required) but would also give  $\dots + 9$ , and so I subtracted 1 to make it  $\dots + 8$ ."

This technique is called *Completing the Square*, as we create a perfect square  $(x + a)^2$ , and then complete it with a constant term, to construct an equivalent expression.

### 7.1 Can you use your knowledge – 6?

1. Which perfect square has a twin that looks like,

- |                        |                        |                        |
|------------------------|------------------------|------------------------|
| 1. $x^2 + 10x + \dots$ | 4. $y^2 + 18y + \dots$ | 7. $x^2 + 5x + \dots$  |
| 2. $x^2 - 10x + \dots$ | 5. $t^2 - 4t + \dots$  | 8. $x^2 - 3x + \dots$  |
| 3. $x^2 + 2x + \dots$  | 6. $k^2 - 30k + \dots$ | 9. $x^2 + 2ax + \dots$ |

2. Complete the right hand side to make these expressions equivalent

- |                                       |  |
|---------------------------------------|--|
| 1. $x^2 + 8x + 7 = (x + 4)^2 \dots$   | 5. $y^2 - 20y = (y - 10)^2 \dots$            |
| 2. $x^2 - 6x + 3 = (x - 3)^2 \dots$   | 6. $x^2 + x + 1 = (x + \frac{1}{2})^2 \dots$ |
| 3. $x^2 + 14x - 9 = (x + 7)^2 \dots$  | 7. $x^2 - 7x - 1 = (x - 3.5)^2 \dots$        |
| 4. $m^2 - 12m + 50 = (m - 6)^2 \dots$ | 8. $x^2 - 0.8x - 0.2 = (x - 0.4)^2 \dots$    |

3. Use the method of *Completing the Square* to write down a non-identical mathematical twin of the form  $(x + h)^2 + k$  for the following

- |                     |                      |                     |
|---------------------|----------------------|---------------------|
| 1. $x^2 + 4x + 7$   | 5. $x^2 + 5x + 2$    | 9. $x^2 + 3x + 8$   |
| 2. $x^2 + 16x + 20$ | 6. $t^2 + 40t + 700$ | 10. $x^2 - 7x - 10$ |
| 3. $x^2 - 10x - 10$ | 7. $x^2 + 9x + 9$    | 11. $y^2 + y - 2$   |
| 4. $x^2 - 2x + 5$   | 8. $y^2 + 8y$        | 12. $x^2 - 11x - 5$ |

This technique can be extended so that it applies to *all quadratic trinomials* by the careful use of the distributive law e.g.

$-x^2 + 3x - 1$	$2x^2 + 8x - 7$
$-(x^2 - 3x) - 1$	$= 2(x^2 + 4x) - 7$
$-[(x - \frac{3}{2})^2 - \frac{9}{4}] - 1$	$= 2[(x + 2)^2 - 4] - 7$
$-(x - \frac{3}{2})^2 + \frac{9}{4} - 1$	$= 2(x + 2)^2 - 8 - 7$
$-(x - \frac{3}{2})^2 + \frac{5}{4}$	$= 2(x + 2)^2 - 15$

4. Use the method of *Completing the Square* to write down a non-identical mathematical twin of the form  $a(x + h)^2 + k$  for the following,

- |                      |                      |                       |
|----------------------|----------------------|-----------------------|
| 1. $2x^2 + 16x + 3$  | 5. $-x^2 + 14x + 10$ | 9. $-3x^2 + 33x + 8$  |
| 2. $3x^2 + 30x - 11$ | 6. $-2x^2 - 24x + 3$ | 10. $5x^2 - 45x + 20$ |
| 3. $2x^2 - 8x$       | 7. $-x^2 - 2x - 5$   | 11. $2x^2 - 3x + 2$   |
| 4. $5x^2 - 30x + 62$ | 8. $4x^2 - 12x + 1$  | 12. $2x^2 + 5x - 3$   |

## 8. The magic of Algebra!

### **8.1 EAT 10**

Think of any odd number.

Square it.

Write down the number one less than this square number.

Repeat for 4 other odd numbers.

What property do the 5 numbers that you have written down have in common?

Can you prove that the result of this process will always share this property?

Number tricks such as the one in EAT 10 can often be analysed using algebraic identities like those you have been studying.

You should have seen that if you

- Think of any odd number.
- Square it.
- Write down the number one less than this square number.

You always get a *multiple of 4*, or at least it seems that way.

The question is whether or not this result could be proved for the *infinite* number of odd numbers that we could think of.

If we *let*  $x$  represent any integer (some write this as  $x \in \mathbf{R}$ ).

Then the expression  $2x + 1$  represents *all odd integers*.

The square of this can now be written as a number of non-identical twins

$$\begin{aligned}(2x+1)^2 \\ &= 4x^2 + 4x + 1 \\ &= 4(x^2 + x) + 1\end{aligned}$$

This last twin was carefully chosen to show that this square of any odd number is *always one greater than a multiple of 4!*

## 8.2 Can you use your knowledge – 5?

For these number tricks, do them three or four times so that you see what is going on, then prove it to be so for all cases...!

1. Choose any integer and write it down.  
Write down its square.  
Now write down the product of the integers "either side" your first number.

What do you notice about the two numbers that you have written down?  
Can you prove it to be so for all cases?

2. Find the sum of any 5 consecutive integers and write it down.  
Repeat for three or four different sets of 5 consecutive integers.

What property do these values share?  
Prove that is true for all possible sets of 5 consecutive integers.

3. Choose any integer.  
Multiply it by the integers "either side" of it.  
Add your original integer.  
Write down any ten consecutive integers that include this result.


Show your maths teacher and be amazed when they tell you what your original integer was!

4. List the square numbers (starting with zero).  
Now list the gaps between consecutive square numbers.

What do you notice?  
Prove it to be so.

## 9. eTech Support.

### 9.1 Copying files from storage memory

Enter the  mode of your CASIO 9860G AU and then press **S MEM** **[F2]** to enter the Storage Memory. Open the **10ALG'ID** folder by moving the (black) input bar onto it (if necessary) and then pressing **[EXE]**.

Move the input bar to the desired file, select **SEL** **[F1]** and **COPY** **[F2]** it.

Its contents will now be copied into the appropriate part of the Main Memory.

These copied files can be deleted from the Main Memory when they are finished with (to make room for new files) and the Storage Memory files will remain for later use.



### Opening a geometry file and animating it.

Enter the **GEOM** mode of your 9860.

Press **[F1]** **File** and then press **z:Open**.

Move the input bar onto the file required and press **[EXE]**.

Geometry File List	
<b>DISTA1</b>	<b>724</b>
DISTA2	1664
DISTA3	2340
DISTB1	724
DISTB2	2300
DISTB3	3024↓

The bottom bar contains **DEL**.

Should you get it, do not worry about the message **Clear Current Image?** – unless there are unsaved changes to a currently open Geometry file that you do not wish to lose.

With a file that is able to be animated (like the ones in this unit), the sequence required to run an animation is,

- go to the **Animate** menu by pressing **[F6]**,
- chose either
  - **5:Go (once)** – for a single “run through” or
  - **6:Go (repeat)** – for repeated animation (stopped by **[AC/ON]**)



## 9.2 Tabulating an expression in a spreadsheet.

Enter the  mode of your 9860. Open a new spreadsheet by pressing File **F1** and New **F1**. Give your spreadsheet an appropriate name.

Type A, B, C and D into the first four cells in row 1.

Type  $(A+B)(C+D)$  into the fifth cell in row 1 (i.e. E1).

Type your non-identical mathematical twin in F1.



SHEET	C	D	E	F	
1	A	B	C	D	$(A+B)(C+D)$
2					
3					
4					
5					$"(A+B)(C+D)"$

To enter text in a 9860 spreadsheet start with a quotation mark ", obtained by pressing

**ALPHA** then **EXP**. Use **ALPHA** or **(A)-LOCK** to enter the red alpha text.

To fill the first 100 cells of columns with random integers between -9 to 9 use the command FILL.

It is found by pressing EDIT **F2** then **F6** then FILL **F1**.



Fill
Formula :=Int (19Ra
Cell Range:A2:D101
EXEC

Enter the formula  $=\text{Int}(19\text{Ran}\#)-10$  with a range of A2:D101.

Int is obtained by pressing **OPTN** then **F6** then NUM **F1** then Int **F2**.

Ran# is obtained by pressing **OPTN** then PROB **F5** then Ran# **F4**.

To compute the value of the expression  $(A+B)(C+D)$  for the 100 sets of randomly chosen A, B, C and D values, use the FILL command for column E with the formula  $=(A2+B2)\times(C2+D2)$  over a range from E2 to E101.

Use a similar method to enter your non-identical mathematical twin in column F and then compare the two columns.

Your 'finished product' should look something like this,

If you wish to save your spreadsheet you will find this command in the File menu, called SV-AS **F3**.



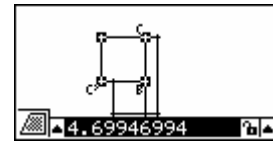
SHEET	B	C	D	E
1	B	C	D	$(A+B)(C)$
2	-7	-8	-5	0
3	-2	6	5	-22
4	2	-10	-2	12
5	-5	-7	3	52

### 9.3 Measuring aspects of geometric constructions.

Any aspect of a geometric construction can be measured by the 9860.

All you need to do is select the features that define the thing you want to measure.

For example, if you want to measure the area of this square you need to select (using the arrow pad and **EXE**) its vertices.



With your selection made, open the Measurement Box by pressing **VAR** and the measurement will be displayed.

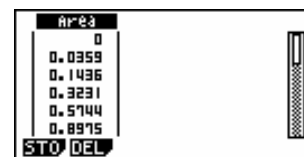
#### Note:

- Press **EXIT** to exit the Measurement Box.
- Press **AC/ON** to deselect all.
- If the Measurement Box is blank you have not selected the features needed to define the thing you wish to measure. Try again.

### Tabulating measured quantities.

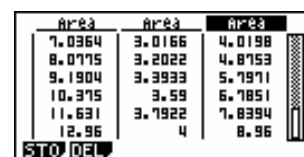
If you have previously run an animation that involved the thing you have measured, then the values of this (and every other) aspect of the construction is stored for each 'step' of the animation

With the defining features of measurement of interest selected (i.e. vertices of the square) go to the Animate menu **F6** and press 7: Add Table.



This table can be added to, so that a number of quantities can be compared.


Press **EXIT** to return to the Geometry window, deselect the selected features (press **AC/ON**) and select the defining features of another aspect of the animation.



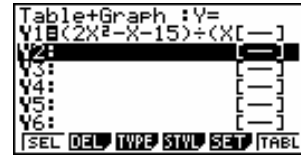
Press 7: Add Table in the Animate menu **F6** to add to your table. In this fashion the three related areas in **DSQUARB4** were compared (above).


The **STO** command **F1** can be used to store the lists of this table in either a LIST in **STAT** mode or in **S-SMT** mode.

## 9.4 Tabulating an expression involving a single variable.

One of the easy ways to look at the values that an algebraic expression takes for a range of inputs is by using the  mode of a 9860.

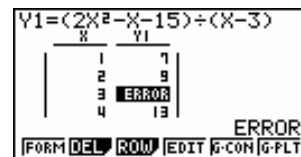
Enter the expression into  $y1$  as  $(2x^2 - x - 15) \div (x - 3)$ .



Press SET  to set the Table Settings.

Choose appropriate values and press .

Press TABL  to see the table of values displayed.





### Plotting the values of an expression.

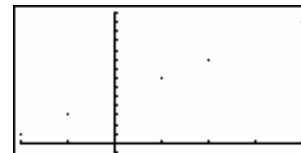
To set the View Window for a scatter plot of the table, press  and V-WIN .





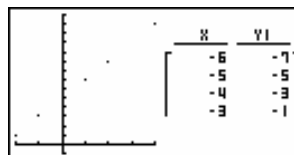
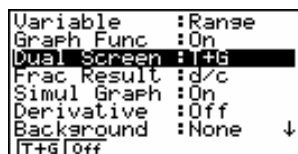
Given the values seen in our table the settings seen left could be used in the drawing of a scatter plot.

Press  and then re-draw the table.

Press G-PLT  to plot the table values on your chosen view window.



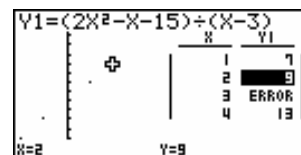
To see table and scatter plot together go to the SET UP by pressing  then  and change your Dual Screen setting to T+G.



You will need to re-draw your table and G-PLT the values once more.

For a finishing touch, press  and GLINK  to link the entries of the table and their corresponding points.

Now move up and down, through your table and plot as one!



## 10. Answers.

### 1.4 Can you use your knowledge - 1?

2.1	$2(3x+1)$
2.2	$4(x+3)$ or $2(2x+6)$
2.3	$10(x+2)$ or $5(2x+4)$ or $2(5x+10)$
2.4	$3(x-4)$
2.5	$5(2x-25)$
2.6	$2(x+\frac{5}{2})$ or $4(\frac{1}{2}x+\frac{5}{4})$ or ...
2.7	$9(x+\frac{4}{9})$ or $4(\frac{9}{4}x+1)$ or ...
2.8	$-2(x+3)$ or $2(-x-3)$ or $-(2x+6)$
2.9	$-3(2x-3)$ or $3(-2x+3)$ or $-(6x-6)$
2.10	$8(4-x)$ or $4(8-2x)$ or $2(16-4x)$
2.11	$-5(1+3x)$ or $5(-1-3x)$ or $-(5+15x)$

3.1	$2x+2y$
3.2	$16x-24$
3.3	$xy+5x$
3.4	$x^2+2x$
3.5	$-4x+20$
3.6	$2x^2+4xy$
3.7	$-x^2+6x$
3.8	$5x^2y-5x^2$

3.9	$6x^3-9x^2y$
3.10	$a^2b+2ab^2$
3.11	$-45x^2y^3+5y^4$

4	$2m+2n+4$ $2(m+1)+2(n+1)$ $(m+2)(n+2)-mn$ $m+1+n+1+$ $+m+1+n+1$
---	---

5.1	$2x(x-4)$ $x(2x-8)$ $2(x^2-4x)$ $x^2-8x+x^2$ $8(x^2+x)-6x^2$
-----	--

5.2	$8x^2+20xy$ $2(4x^2+10xy)$
-----	-------------------------------

5.3	$2(5-9x^2)$ $10(1-1.8x^2)$
-----	-------------------------------

5.4	$3z(y-8x)$ $z(3y-24x)$ $3(yz-8xz)$
-----	--

5.5	$x^2(18-23y)$ $x(18x-23xy)$
-----	--------------------------------

5.6	$3x^2-3xy+9x$ $3(x^2-xy+3x)$
-----	---------------------------------

5.7	$2x^2+3xz-5x$ $x^2+x^2+xy+$ $+xy+xy-5x$
-----	---

5.8	$4(3y+y^2)$ $4y(3+y)$ $y(12+4y)$
-----	--

5.9	$a^2b+ab^2$ $a(ab+b^2)$ $b(a^2+ab)$
-----	---

5.10	$x^2+7x-4x$ $x^2+3x$ $x(x+7-4)$ $x(x+3)$
------	---

5.11	$10-x^2-3x$ $10-x^2-x-x-x$
------	-------------------------------

5.12	$2x-10+xy+3x$ $x(y+5)-10$
------	------------------------------

5.13	$(x-4)(y+3)$ $x^2-4x+7x-28$ $x^2+3x-28$
------	---

5.14	$(1-z)(3+y)$ $3-3z+y-yz$
------	-----------------------------

5.15	$zx-4z-8z+2xz$ $zx-12z+2xz$ $z(x-4)+2z(x-4)$ $3z(x-4)$ $3xz-12z$
------	--

### 2.3 Can you use your knowledge - 2?

2.1	$x^2+14x+49$
-----	--------------

2.2	$y^2+6y+9$
-----	------------

2.3	$x^2+2x+1$
-----	------------

2.4	$z^2-20z+100$
-----	---------------

2.5	$x^2-10x+25$
-----	--------------

2.6	$4x^2+4x+1$
-----	-------------

2.7	$y^2-12y+36$
-----	--------------

2.8	$25x^2+40x+16$
-----	----------------

2.9	$64-16w+w^2$
-----	--------------

2.10	$25i^2+20ij+4j^2$
------	-------------------

2.11	$16a^2-24ab+9b^2$
------	-------------------

2.12	$36x^2-132xy+121y^2$
------	----------------------

3	1, 4, 6, 9
---	------------

4.1	$4x$
4.2	$20x$
4.3	$81$
4.4	$36$
4.5	$x^2$
4.6	$24k$
4.7	$24xy$
4.8	$36x^2$
4.9	$14p^2q$

5.1	$x^2 - 10x + 25$
5.2	$y^2 + 18y + 81$
5.3	$a^2 - 2ab + b^2$
5.4	$9k^2 + 12km + 4m^2$
5.5	$25p^2 + 50pq + 25q^2$
5.6	$16x^2 - 48xy^2 + 36y^4$

6.1	Only one answer
6.2	$y^2 + 9y + \frac{81}{4}$
6.3	$a - 2\sqrt{ab} + b^2$
6.4	$3k^2 + 4\sqrt{3}km + 4m^2$
6.5	$25p^2 + 10p + 1$
6.6	$16x^2 - 48xy + 36y^2$

7.1	$(y+10)^2$
7.2	$(x-4)^2$
7.3	$(k-3)^2$
7.4	$(5+m)^2$
7.5	$(x+y)^2$
7.6	Not possible
7.7	Not possible
7.8	$(2a+6)^2$
7.9	$(3x-4y)^2$
7.10	$(5p+8q)^2$
7.11	$(x^2-9y)^2$
7.12	Not possible

**3.2 Can you use your knowledge - 3?**

2.1	$x^2 + 5x + 2x + 10$ $x^2 + 7x + 10$
2.2	$y^2 - 6y - 7y + 42$ $y^2 - 13y + 42$
2.3	$x^2 - 15x + x - 15$ $x^2 - 14x - 15$
2.4	$x^2 + px + qx + pq$
2.5	$ab + 3b - 4a - 12$
2.6	$2x^2 - 18x - 3x + 27$ $2x^2 - 21x + 27$
2.7	$150x^2 + 15x - 10x - 1$ $150x^2 + 5x - 1$
2.8	$6y^2 + 8y + 3y + 4$ $6y^2 + 11y + 4$
2.9	$15xy + 12y - 10xz - 8z$
2.10	$x^2 - 6x + 6x - 36$ $x^2 - 36$
2.11	$1 + z - z - z^2$ $16a^2 - 12ab + 12ab - 9b^2$ $16a^2 - 9b^2$

4.1	$(x+1)(x+7)$
4.2	$(x+8)(x+2)$
4.3	$(x+3)(x+4)$
4.4	$(x+5)(x-2)$
4.5	$(x-4)(x+1)$
4.6	$(x-4)(x-7)$
4.7	$(x-5)(x+4)$
4.8	$(x-3)(x-6)$
4.9	$(x+8)(x+3)$
4.10	$(x-4)(x-2)$
4.11	$(x-1)(x-9)$
4.12	$(x-12)(x+3)$

5.1	$(2x+1)(x+3)$
5.2	$(3x+5)(x+1)$
5.3	$(2x+3)(x+2)$
5.4	$(5x-2)(x-2)$
5.5	$(2x+1)(2x-5)$
5.6	$(3x-2)(2x+5)$
5.7	$(10x-7)(x+1)$
5.8	$(x-3)(6x-5)$
5.9	$(4x+1)(x+8)$
5.10	$(6x-7)(2x+3)$
5.11	$(5x+8)(2x-3)$
5.12	$(6x-7)(3x-2)$

**4.3 Can you use your knowledge - 4?**

1	1, 2, 4, 6, 7, 8
2.1	$x^2 - 16$
2.2	$y^2 - 49$
2.3	$4 - x^2$
2.4	$4x^2 - 1$
2.5	$y^2 - 9x^2$
2.6	$25p^2 - 16q^2$
2.7	$x^2 - 36$
2.8	$x^2y^2 - 4z^2$
2.9	$64n^2 - 81p^4$

3.1	$(x-5)(x+5)$
3.2	$(y-2)(y+2)$
3.3	$(3-x)(3+x)$
3.4	$(m-n)(m+n)$
3.5	$(2x-y)(2x+y)$
3.6	$(a-12b)(a+12b)$
3.7	$(3x-6)(3x+6)$ $9(x-2)(x+2)$
3.8	$(4a-7b)(4a+7b)$
3.9	$(9x-10y)(9x+10y)$

4.1	$[x-(x+3)]$ $\times [x+(x+3)]$ $-3(2x+3)$
4.2	$[(y-2)-2y]$ $\times [(y-2)+2y]$ $(-y-2)(3y-2)$ $-(y+2)(3y-2)$
4.3	$[x-(1-x)]$ $\times [x+(1-x)]$ $(2x-1)\times 1$
4.4	$[(x+3)-(x-2)]$ $\times [(x+3)+(x-2)]$ $5(2x+1)$
4.5	$[(x-4)-(x-6)]$ $\times [(x-4)+(x-6)]$ $2(2x-10)$ $4(x-5)$
4.6	$[(2x+3)-(x+1)]$ $\times [(2x+3)+(x+1)]$ $(x+2)(3x+4)$
4.7	$[(x+y)-(x-y)]$ $\times [(x+y)+(x-y)]$ $2y\times 2x$ $4xy$
4.8	$[(2x+y)-(x-2y)]$ $\times [(2x+y)+(x-2y)]$ $(x+3y)(3x-y)$
4.9	$[(m-2n)-(n-m)]$ $\times [(m-2n)+(n-m)]$ $(2m-3n)(-n)$ $-n(2m-3n)$
5.1	$(x-\sqrt{3})(x+\sqrt{3})$

5.2	$(x-\sqrt{10})(x+\sqrt{10})$
5.3	$(\sqrt{8}-y)(\sqrt{8}+y)$ $(2\sqrt{2}-y)(2\sqrt{2}+y)$
5.4	$(m-\sqrt{2n})(m+\sqrt{2n})$
5.5	$(\sqrt{5x}-\sqrt{11y})$ $\times (\sqrt{5x}+\sqrt{11y})$
5.6	$(a-\sqrt{b})(a+\sqrt{b})$
5.7	$(\sqrt{7x}-\sqrt{3w^2})$ $\times (\sqrt{7x}+\sqrt{3w^2})$
5.8	$(a\sqrt{b}-c)(a\sqrt{b}+c)$
5.9	$[\sqrt{12x}-\sqrt{3x+2}]$ $\times [\sqrt{12x}+\sqrt{3x+2}]$ $(2\sqrt{3x}-\sqrt{3}-2)$ $\times (2\sqrt{3x}+\sqrt{3}+2)$ $(\sqrt{3x}-2)(3\sqrt{3x}+2)$

**5.1 Can you use your knowledge - 5?**

1.1	$2x+6y$
1.2	$y^2-2y-15$
1.3	$-x^2-8x-16$
1.4	$-2x^2+3x$
1.5	$-5z^2+500$
1.6	$18x^2-21x-15$
1.7	$-4y^3+34y^2-60y$
1.8	$-12x^3+108x^2$ $-243x$
1.9	$w^2-5w+19$
1.10	$10i+4j-3ij$
1.11	$a^4b^2-9b^2$
1.12	$10x-5$
2.1	$5x(x-4)$

2.2	$-(x+2)^2$
2.3	$(z+2)(z+8)$
2.4	$-x(3x-8)$
2.5	$(4x+y)^2$
2.6	$-(x+10)(x-10)$
2.7	$4xy(2+3y)$
2.8	$3(5x-2y)$ $\times (5x+2y)$
2.9	$-3(t-1)(t-8)$
2.10	$-2x(x-2y)$ $\times (x+2y)$
2.11	$(2a+6)^2$
2.12	$-2x(4x-2)$

**6.2 Can you use your knowledge - 6?**

1.1	$\frac{4(2x-1)}{2x-1}$ $= 4$ if $x \neq \frac{1}{2}$
1.2	$\frac{5(2x+4)}{2x+4}$ $= 5$ if $x \neq -2$
1.3	$\frac{3(8x+6)}{8x+6}$ $= 3$ if $x \neq -\frac{3}{4}$
1.4	$\frac{3(3y-5)}{4(3y-5)}$ $= \frac{3}{4}$ if $y \neq \frac{5}{3}$
1.5	$\frac{x(x+5)}{x}$ $= x+5$ if $x \neq 0$
1.6	$= 7-2x$ if $x \neq 0$
1.7	$= x^2-3$ if $x \neq 0$
1.8	$\frac{(x+1)(x-1)}{x-1}$ $= x+1$ if $x \neq 1$

1.9	$\frac{(x+1)(x+4)}{x+1}$ $= x+4$ if $x \neq -1$
1.10	$\frac{(x-2)(x-5)}{x-2}$ $= x-5$ if $x \neq 2$
1.11	$\frac{(2x-3)^2}{2x-3}$ $= 2x-3$ if $x \neq \frac{3}{2}$
1.12	$\frac{(2x-5)(x+4)}{x+4}$ $= 2x-5$ if $x \neq -4$

2.1	$\frac{(x+7)(x-2)}{x(x+7)}$ $= \frac{x-2}{x}$ if $x \neq -7$
2.2	$\frac{(x+4)(x-2)}{(x+4)(x+1)}$ $= \frac{x-2}{x+1}$ if $x \neq -4$
2.3	$\frac{(x-2)^2}{(x-2)(x+2)}$ $= \frac{x-2}{x+2}$ if $x \neq 2$
2.4	$\frac{x(x+1)}{(1-x)(1+x)}$ $= \frac{x}{1-x}$ if $x \neq -1$
2.5	$\frac{(x+8)(x-2)}{3(x+8)}$ $= \frac{x-2}{3}$ if $x \neq -8$
2.6	$\frac{(2x-1)(x-3)}{(x-3)^2}$ $= \frac{2x-1}{x-3}$ if $x \neq 3$

2.7	$\frac{2(x-3)(x+5)}{4x(x-3)}$ $= \frac{x+5}{2x}$ if $x \neq 3$
2.8	$\frac{-(x^2-x+1)}{x^2-x+1}$ $= -1$ as $x^2-x+1 \neq 0$
2.9	$\frac{9x(x-2)}{9(x-2)(x+2)}$ $= \frac{x}{x+2}$ if $x \neq 2$
2.10	$\frac{(2x-5)(3x-2)}{(2x-5)^2}$ $= \frac{3x-2}{2x-5}$ if $x \neq \frac{5}{2}$
2.11	$\frac{a(x-y)}{b(x+y)(x-y)}$ $= \frac{a}{b(x+y)}$ if $x \neq y$
2.12	$\frac{(4x-y)(4x+y)}{y(y-4x)}$ $= \frac{-(4x+y)}{y}$ if $y \neq 4x$

**7.1 Can you use your knowledge - 7 ?**

1.1	$(x+5)^2$
1.2	$(x-5)^2$
1.3	$(x+1)^2$
1.4	$(y+9)^2$
1.5	$(t-2)^2$
1.6	$(k-15)^2$
1.7	$(x+\frac{5}{2})^2$
1.8	$(x-\frac{3}{2})^2$
1.9	$(x+a)^2$

2.1	-9
2.2	-6
2.3	-58
2.4	+14
2.5	-100
2.6	$+\frac{3}{4}$
2.7	$-\frac{53}{4}$
2.8	-0.36

3.1	$(x+2)^2+3$
3.2	$(x+8)^2-44$
3.3	$(x-5)^2-35$
3.4	$(x-1)^2+4$
3.5	$(x+\frac{5}{2})^2-\frac{17}{4}$
3.6	$(t+20)^2+300$
3.7	$(x+\frac{9}{2})^2-\frac{45}{4}$
3.8	$(y+4)^2-16$
3.9	$(x+\frac{3}{2})^2+\frac{23}{4}$
3.10	$(x-\frac{7}{2})^2-\frac{89}{4}$
3.11	$(y+\frac{1}{2})^2-\frac{9}{4}$
3.12	$(x-\frac{11}{2})^2-\frac{141}{4}$

4.1	$2(x+4)^2-29$
4.2	$3(x+5)^2-86$
4.3	$2(x-2)^2-8$
4.4	$5(x-3)^2+17$
4.5	$-(x-7)^2+59$
4.6	$-2(x+6)^2+75$
4.7	$-(x-1)^2-4$
4.8	$4(x-\frac{3}{2})^2-8$
4.9	$-3(x-\frac{11}{2})^2+\frac{395}{4}$
4.10	$5(x-\frac{9}{2})^2-\frac{325}{4}$
4.11	$2(x-\frac{3}{4})^2+\frac{7}{8}$
4.12	$2(x+\frac{5}{4})^2-\frac{49}{8}$