The In-betweens

Not-whole numbers and their various forms

Booklet 5 (of 5)

In this booklet:

- **Adding and subtracting numbers in decimal form**
- **Multiplying and dividing by powers of 10**
- **Multiplying and dividing numbers in decimal form**
- **Rounding numbers in decimal form**
- **Not-so-*nice* calculations**
- **More infinite thingies**

Student Name: ____

Teacher Name: ____

Class: ____

Commencement date: ____

Smooth Mathematics

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The In-Betweens
Not-whole numbers and their forms

Booklet 5 (of 5)

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Created by Anthony and Tayla Harradine with the help of many wonderful people.

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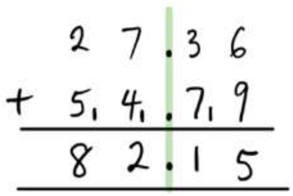
Contents

23. Operations involving numbers in decimal form

23.1 Adding numbers in decimal form

The algorithm for adding *non-whole numbers* in decimal form works the same way as the algorithm for adding whole numbers.

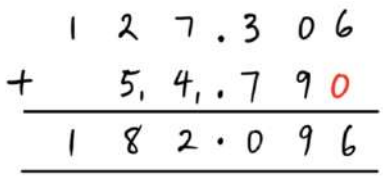
For example, to find the result of $27.36 + 54.79$ we do the following.

	Be sure to vertically align the digits with the same place value (and so the decimal point). Working from *right to left*, the 9 and 6 represent *nine* $\frac{1}{100}$s and *six* $\frac{1}{100}$s respectively. So, adding gives $\frac{15}{100}$. But we cannot write 15 in the $\frac{1}{100}$s place. Since $\frac{15}{100} = \frac{10}{100} + \frac{5}{100} = \frac{1}{10} + \frac{5}{100}$, we "carry" the 1 (tenth) into the $\frac{1}{10}$s place, write 5 in the $\frac{1}{10}$s place, and continue. Now, • $3 + 7 + 1 = 11$ (tenths), carry the 1 to the units (ones) place. • $7 + 4 + 1 = 12$ (units/ones), carry the 1 (ten) to the tens place. • $2 + 5 + 1 = 8$ (tens). Done, 82.15

Example 1.

Calculate

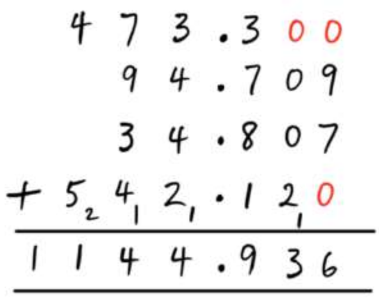
127.306 + 54.79



Example 2.

Calculate

437.3 + 94.709 +34.087+542.12



Remember, carefully align (vertically) the digits with the same place value.
Also, if you like, insert trailing-zeros so that each number has the same number of decimal places filled.

Question 1

Calculate each of the following using the addition algorithm.

a) 12.45 + 0.32

b) 35.25 + 0.48

c) 28.45 + 91.26

d) 57.45 + 5.014

e) 89.45 + 34.107

f) 64.045 + 59.66

Question 2

Calculate each of the following using the addition algorithm.

a) $352.18 + 54.11 + 0.052$

b) $145.27 + 153.043 + 27.509 + 11.011$

c) $42.4 + 973.421 + 145.027 + 11.011 + 4.5608$

Question 3

Calculate, using whatever method comes to mind.

a) $3.2 + 0.9$

b) $0.3 + 0.9$

c) $1.5 + 1.6$

d) $4.8 + 4.7$

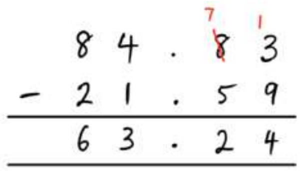
e) $7.8 + 5.4$

f) $14.7 + 6.5$

23.2 Subtracting numbers in decimal form

The subtraction algorithm for subtracting one *non-whole number* from another, in decimal form, works the same way as the algorithm for subtracting one whole number from other.

For example, to find the result of 84.83 – 21.59 we do the following:

<table>
<tr>
<td>
</td>
<td>Working from right to left, 9 subtracted from 3 cannot be done, given we must stick to positive numbers.

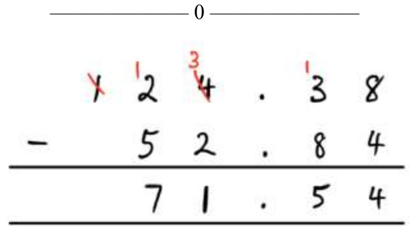
So, we “move” $\frac{1}{10}$ of the $\frac{8}{10}$s, from the tenth’s place to the hundredth’s place, leaving seven-tenths.
Since $\frac{1}{10} = \frac{10}{100}$, we now have $\frac{13}{100}$s.
We now subtract,
• 13 – 9 = 4 (hundredths)
• 7 – 5 = 2 (tenths)
• 4 – 1 = 3 (units/ones)
• 8 – 2 = 6 (tens).

Done.
84.83 – 21.59 = 63.24</td>
</tr>
</table>

Example 1.

Calculate

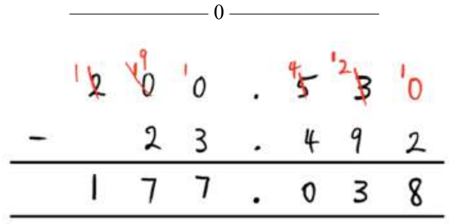
124.38 – 52.84



Example 2.

Calculate

200.53 – 23.492



Take care to carefully align (vertically) the corresponding place values and insert trailing-zeros so that each number has the same number of decimal places filled.

Question 1

Calculate each of the following using the subtraction algorithm.

a) 82.45 – 51.35

b) 324.45 – 51.41

c) 378.97 – 181.54

d) 725.16 – 515.54

e) 374.07 – 285.54

f) 64.045 – 59.66

Question 2

Calculate each of the following using the subtraction algorithm.

a) 123.45 – 67.89
b) 427.304 – 68
c) 30000.4 – 128.06
d) 57 – 5.01
e) 712.014 – 545.67
f) 984 – 732.679

Question 3

Calculate, using whatever method comes to mind.

a) 12 – 4.2
b) 20 – 16.3
c) 4 – 0.9
d) 7 – 2.8
e) 103.6 – 4.7
f) 0.0042 – 0.0007

23.3 Multiplying and dividing a number in decimal form by 10 or 100 or …

What is the result of 672×10?

Easy?
$672 \times 10 = 6720$.
Just "add a 0".

"Add a 0", of course, does *not* mean $+0$.
"Add a 0" means write a 0 on the right-hand end of the number.

> It is important to know that "add a zero" is *the same* as "move the decimal point one place to the right".
> Knowing that $672 = 672.0$, helps to appreciate why.
>
> Technical cases exist when we would write 672.0, but it is most common that whole numbers are written without the decimal place.

What is the result of 401×1000?

Well, $1000 = 10 \times 10 \times 10$. So, we could multiply by 10 three times.
Hence, we could move the decimal point *three places to the right*.
It might help you to write in some helping-0s, as we have done (in red).

$$401.000 \times 1000 = 401000$$

What is the result of 34.89×1000?

$$34.890 \times 1000 = 34890$$

> If a number is multiplied (×) by 10^n, move the decimal point *n* places to the right.

What about division? Move left! ☺

> If a number is divided (÷) by 10^n, move the decimal point *n* places to the left.

We will explain why this works in the next section. ☺

What is the result of $245 \div 10000$?
Move decimal point 4 places to the left. But we only have three digits, so write down a second helping-0, in the thousands place.)

$$0245.0 \div 10000 = 0.0245$$

It is a convention to include the leading zero when the *result* is less than 1.
It is also a convention, in most cases*, not to include a 0 after the 5 (the right-most digit).

* It would make sense to write 0.02450, if this number represented a measurement and we had *measured* the number of $\frac{1}{100000}$ s to be 0.

Example 1.

Calculate,

$$0.4 \times 10$$

0

$$0.4 \times 10$$

$$= 4 \qquad (0.4)$$

Example 2.

Calculate,

$$0.00052 \times 100$$

0

$$0.00052 \times 100$$

$$= 0.052 \qquad (0.00052)$$

Example 3.

Calculate,

$$24.38 \times 1000$$

0

$$24.38 \times 1000$$

$$= 24380 \qquad (24.380)$$

Example 4.

Calculate,

$$20000.53 \div 100$$

0

$$20000.53 \div 100$$

$$= 200.0053 \qquad (20000.53)$$

Example 5.

Calculate,

$$\frac{743287}{100}$$

0

$$\frac{743287}{100}$$

$$= 743287 \div 100$$

$$= 7432.87$$

Example 6.

Calculate,

$$\frac{57}{100000}$$

0

$$\frac{57}{100000}$$

$$= 57 \div 100000$$

$$= 0.00057 \qquad (0.00057.0)$$

Question 1

Calculate each of the following.

a) 8.42×10

b) $24.38 \div 10$

c) 0.7×100

d) $24.38 \div 100$

e) 0.4×1000

f) $24.38 \div 1000$

Question 2

Calculate each of the following.

a) 12.47×1000

b) $24850 \div 1000$

c) 0.000452×100

d) $4.57 \div 100$

e) 20.0004×10

f) $0.034 \div 10$

Question 3

Calculate each of the following.

a) 1.728×10^8

b) $20000000 \div 10^{10}$

c) 0.0000402×10^3

d) $2.01 \div 10^5$

e) $10^2 \times 10^4$

f) $10^2 \div 10^4$

Question 4

What number should each *not-whole number* be multiplied by so the result is the smallest whole number possible.

a) 23.561

b) 0.21

c) 4284.0222

d) 3.1

e) 14.0000000001

f) 2000.01002

23.4 Why "moving the decimal point" works

"Moving the decimal point" is a very convenient (and accurate) way to go about multiplying and dividing by 10^n. However, *moving the decimal point* is not really what is going on!
So, what is going on?

438.92 is a not-whole number written in decimal form and so,

$$438.92 = 4 \text{ hundreds} + 3 \text{ tens} + 8 \text{ ones} + 9 \text{ tenths} + 2 \text{ hundredths}$$
$$= (4 \times 100) + (3 \times 10) + (8 \times 1) + \left(9 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right)$$

To multiply 438.92 by 10, we could multiply each of the parts that *make up* 438.92 by 10 and then add the results together.
You might know this idea by the name, *the distributive law*.
The idea was discussed in an earlier booklet.
It works like this,

$$438.92 \times 10$$
$$= (4 \times 100 \times 10) + (3 \times 10 \times 10) + (8 \times 1 \times 10) + \left(9 \times \frac{1}{10} \times 10\right) + \left(2 \times \frac{1}{100} \times 10\right)$$

Now, using our knowledge (how and why) about multiplying powers of 10 by 10 and our know of how to multiply fractions by whole numbers:

$$438.92 \times 10$$
$$= (4 \times 1000) + (3 \times 100) + (8 \times 10) + (9 \times 1) + \left(2 \times \frac{1}{10}\right)$$
$$= 4389.2$$

The result of the multiplication is that:

- the 4 *hundreds* become 4 *thousands*
- the 3 *tens* become 3 *hundreds*
- the 8 *ones* become 8 *tens*
- the 9 *tenths* become 9 *units* and
- the 2 *hundredths* become 2 *tenths*

So, when multiplying a number in decimal form by 10,

"each digit moves to the next highest place value".

The *same result* can be achieved by "*moving the decimal point one place to the right*".

This result applies when multiplying any number in decimal form by 10, non-whole or whole.

We can reason in a similar way to make sense of the fact that when dividing a number in decimal form by 10,

"each digit moves to the next lowest place value".

The *same result* can be achieved by "*moving the decimal point one place to the left*".

This result applies when multiplying any number in decimal form by 10, non-whole or whole.

Can you see what is really going on when we choose to move the decimal point 2 places to the right when multiplying by 100?

Question 1

Do your best to explain, in your own words, what is *really going on* when we calculate the result of 34.562 × 1000 by moving the decimal point three places to the right.

Question 2

Do your best to explain, in your own words, what is really going on when we calculate the result of $34.562 \div 100$ by moving the decimal point two places to the left.

23.5 Multiplying numbers in decimal form

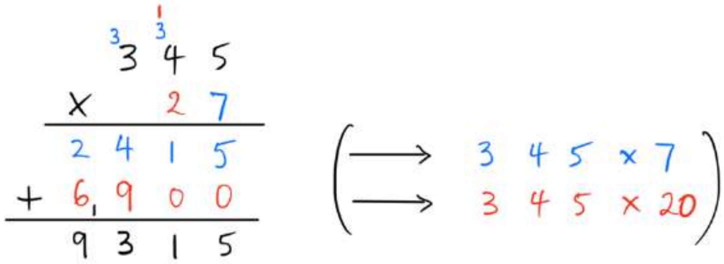
What is the result of 2.4×0.25?

Well, $0.25 = \frac{1}{4}$ and so $2.4 \times 0.25 = 2.4 \times \frac{1}{4}$ and one-quarter of 2.4 is 0.6.
Not all calculations of this type can be handled easily using an approach like this.

You most likely know how to multiply one whole number in decimal form by another.
You may also know more than one method.

Let's calculate 345×27.

We will focus on using the *long multiplication* algorithm in this section.
But before starting, an estimate the result suggests it will be about 9000 (300×30).
The algorithm works like this,



The finer details of how this algorithm works were discussed in an earlier booklet.

We can use the long multiplication algorithm to multiply one not-whole number in decimal form by another.
But we need to include two extra steps.

Suppose we need to calculate 3.45×2.7.
Estimating suggests the result should be about 9 (3×3).
Here is how we get the exact result:

- First, multiply each number by a power of 10 so that each becomes a whole number,
$$(3.45 \times 100) \times (2.7 \times 10) = 345 \times 27$$
- Then calculate 345×27, using the long multiplication algorithm (as seen above), to get 9315.
- But 9315 is *not* the result we want. It is too big, but we can correct that.
  9315 is $\times\ 100$ and $\times\ 10$ too large, or more simply $\times\ 1000$ too large.
  So, we need to divide 9315 by 1000,
$$9315 \div 1000 = 9.315$$

So, $3.45 \times 2.7 = 9.315$

Written out by-hand, the calculation looks like this,

$$\begin{aligned} & 3.45 \times 2.7 \\ = \; & 345 \times 27 \div 1000 \\ = \; & 9315 \div 1000 \\ = \; & 9.315. \end{aligned}$$

$$\begin{array}{r} {}^{3}3\,{}^{1}_{3}4\,5 \\ \times \quad 2\,7 \\ \hline 2\,4\,1\,5 \\ +\;6_{1}\,9\,0\,0 \\ \hline 9\,3\,1\,5 \end{array}$$

Example 1.

Calculate 31.2 × 4.3

——— 0 ———

Result $\approx 30 \times 4 = 120$

$$\begin{aligned} & 31.2 \times 4.3 \\ = \; & 312 \times 43 \div 100 \\ = \; & 13416 \div 100 \\ = \; & 134.16 \end{aligned}$$

$$\begin{array}{r} 3\,1\,2 \\ \times \quad 4\,3 \\ \hline 9\,3\,6 \\ +\,1\,2_{1}\,4_{1}\,8\,0 \\ \hline 1\,3\,4\,1\,6 \\ \hline \end{array}$$

Example 2.

Calculate 4315.3 × 0.03

——— 0 ———

Result $\approx 4000 \times 3 \div 100 = 120$

$$\begin{aligned} & 4315.3 \times 0.03 \\ = \; & 43153 \times 3 \div 1000 \\ = \; & 129459 \div 1000 \\ = \; & 129.459 \end{aligned}$$

$$\begin{array}{r} 4\,3\,{}^{1}1\,5\,3 \\ \times \quad 3 \\ \hline 1\,2\,9\,4\,5\,9 \\ \hline \end{array}$$

Question 1

Use the long multiplication algorithm to calculate:

a) 270.3×0.6 b) 4018.6×0.09 c) 2.401×0.004

Question 2

Use the long multiplication algorithm to calculate:

a) 315.3×2.8 b) 0.88×9.1 c) 0.00341×6.4

Question 3

Calculate, using whatever method that comes to mind.

a) 5×0.2

b) 3.2×0.5

c) 1.2×0.25

d) 32×0.125

e) 50×0.6

f) $99 \times 0.\overline{3}$

Question 4

Calculate, using whatever method that comes to mind.

a) 12×3.5

b) 15×1.2

c) 40×6.25

d) 0.9×1.0007

e) 0.041×2.006

f) $0.1\overline{6} \times 1.26$

23.6 Dividing numbers in decimal form

You know that $8 \div 4 = 2$ (remainder 0).

You also know that $9 \div 4 = 2$ remainder 1.

You know that $9 \div 4 = \frac{9}{4} = 2\frac{1}{4}$

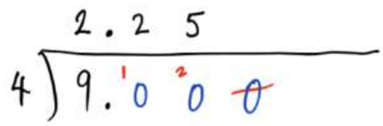
So, in the case,

$$\text{remainder } 1 = \frac{1}{4} = 0.25$$

and that means

$$9 \div 4 = 2.25$$

We can also determine that $9 \div 4 = 2.25$ using the short division algorithm, as you learned in an earlier booklet. In this case,



The reason this process works is illustrated in the "cake" video.
Ask your teacher to show it to you if you do not have access to it.

What if 9 and 4 were *not* whole numbers?
For example, what if we had to calculate $9.35 \div 0.4$?

We can use the short division algorithm, *but we need to include two extra steps*.

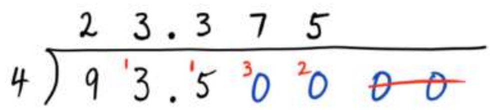
First, we multiply *the divisor* (0.4) by a *power of 10* so that it becomes a whole number.

$$0.4 \times 10 = 4$$

Second, we multiply *the dividend* (9.35) by the *same power of 10* we multiplied the divisor by; *in this case* 10.

$$9.35 \times 10 = 93.5$$

We then calculate $93.5 \div 4$,

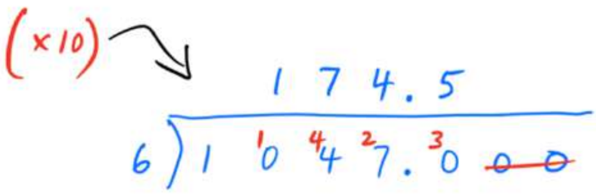


Importantly, the result of 93.5 ÷ 4 will be the same as the result of 9.35 ÷ 0.4. Why?
Because we have multiplied both the dividend and divisor by the same number.
It is just like scaling up a fraction!
So, 9.35 ÷ 0.4 = 23.375
A terminator! ☺

Example 1.

Calculate 104.7 ÷ 0.6

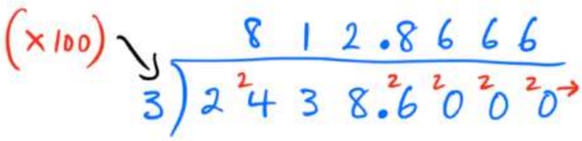
$$104.7 \div 0.6$$
$$= 1047 \div 6$$
$$= 174.5$$



Example 2.

Calculate 24.386 ÷ 0.03

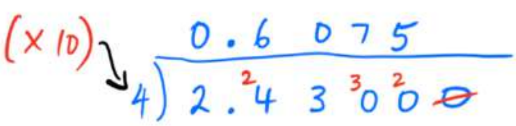
$$24.386 \div 0.03$$
$$= 2438.6 \div 3$$
$$= 812.8\bar{6}$$



Example 3.

Calculate 0.243 ÷ 0.4

$$0.243 \div 0.4$$
$$= 2.43 \div 4$$
$$= 0.6075$$



Question 1

Use the short division algorithm to calculate:

a) $12.6 \div 0.7$ b) $52.45 \div 0.4$ c) $72.46 \div 0.3$

Example 4.

Calculate 724.38 ÷ 2.4

$$724.38 \div 2.4$$
$$= 7243.8 \div 24 \quad (\times 10)$$
$$= \underline{301.825}$$

$$24 \overline{)\,7243.8000} = 301.825$$

24, 48, 72, 96, 120, 144, 168, 192

Example 5.

Calculate 4.8 ÷ 0.4

$$4.8 \div 0.4$$
$$= 48 \div 4$$
$$= \underline{12}$$

Example 6.

Calculate 2 ÷ 1.6

$$2 \div 1.6$$
$$= 20 \div 16$$
$$= \frac{20}{16}$$
$$= \frac{5}{4}$$
$$= 1\frac{1}{4}$$
$$= \underline{1.25}$$

Question 2

Use the short division algorithm to calculate:

a) $0.345 \div 0.6$

b) $0.00123 \div 0.4$

c) $32.45 \div 1.2$

d) $724.38 \div 2.4$

e) $0.02438 \div 0.7$

Question 3

Calculate, using whatever method comes to mind.

a) $4.2 \div 0.6$
b) $36 \div 0.4$
c) $0.72 \div 0.8$
d) $5.2 \div 1.2$
e) $6 \div 1.6$
f) $6.5 \div 0.8$

23.7 Rounding not-whole numbers in decimal form

The image seen right shows a screen from a fitness app.

Three outdoor walks have taken place that covered:

- 0.52 km,
- 5.00 km and
- 3.76 km respectively.

Is 5.00 representing a whole number of kilometres?
If it is, why is the .00 shown?

Before addressing these questions, let's first explain what 3.76 km means. It means,

- 3 whole kms
- + 7 tenths of one km
- + 6 hundredths of one km.

It can be hard to think about "how far" tenths and hundredths of kilometres are, so what are they in metres?

$$3.76 \text{ km} = 3 \text{ km} + 700 \text{ m} + 60 \text{ m}.$$

The first decimal place to the right of the decimal point measures *hundreds of metres*.
The second measures *tens of metres*.
If there was a third decimal place after the decimal point, it would be representing some number of *one metres*.

The app uses a GPS method to *measure* how far a person walks.
The 5.00 tell us that the app is sure of the three digits it displays.
It is sure the person walked 5 whole kilometres, 0 hundreds of metres and 0 tens of metres.
But the app is not sure beyond tens of metres.
The person might have walked 5.004 km, i.e. 5 whole kilometres and 4 metres, but the GPS mode cannot accurately measure single metres.

The .00, in 5.00, is included because it gives us information about the measurement.

In some measurement contexts, the system doing the measuring might result in us being sure about more information than is required.
For example, the scales might report that Joe weighs 77.8 kg, but he might not require that level of detail and may choose to state his weight as 78 kg.
Why 78 and not 77?
Well, 0.8 kg is closer to 1 kg than 0 kg, so it makes sense to round 77.8 up, to 78.

If the scales reported 77.5 kg, maybe Joe would have stated 77.5, or maybe 77 or maybe 78; what would you do? It might depend on the context.

Decimal places

26.786 is a not-whole number in decimal form.
Recall that 26.786 has a *fractional part*, 0.786.
The *digits after the decimal point* are said to occupy the number's ***decimal places***.

So, the number 26.786 is said to have *three decimal places* (or 3 d.p.).

7 is in the *first* decimal place, 8 in in the *second* decimal place and 6 is in the *third* decimal place.

When rounding not-whole number in decimal form, the idea of decimal places is often used. For example, to round 26.78X correct to 2 d.p., we would look at the third decimal place digit (X) and if it is:

- less than 5, the 8 will remain as 8,
- greater than 5, the 8 will go up to 9,
- equal to 5, the 8 will go up to 9.

If X = 6, we would be rounding 26.786 and the 8 would become 9.
So, 26.786 = 26.79 correct to 2 d.p..

It is like asking if 86 is closer to 80 or 90.
We do not write 26.790. If we did, we would be suggesting there are zero thousandths, which is not true.

Rounding correct to *n* d.p.

To round a number correct to n d.p., consider *only* the digit in the $(n+1)^{\text{th}}$ decimal place (the one "next right").

If the digit in the $(n+1)^{\text{th}}$ decimal place is:

- less than 5, *do not change* the digit in the n^{th} decimal place,
- greater than 5, *add one* to the digit in the n^{th} decimal place,
- equal to 5, *add one* to the digit in the n^{th} decimal place.

Example 1.

Round 98.5273999 correct to 3 d.p..

0

Look at the digit in the 4^{th} decimal place.
It is a 3. (98.527**3**999)

3 is less than 5, so the digit in the third decimal place remains the same, i.e., 7.

98.5273999 = 98.527 correct to 3 d.p.

Example 2.

Round 0.00486802 correct to 4 d.p..

0

Look at the digit in the 5^{th} decimal place.
It is a 6. (0.0048**6**802)

6 is greater than 5, so the digit in the 4^{th} decimal place increases by one, i.e., 9.

0.00486802 = 0.0049 correct to 4 d.p.

Example 3.

Round 124.12954 correct to 3 d.p..

0

Look at the digit in the 4th decimal place. It is a 5.

So, the digit in the third decimal place, 9, increases by one, to 10. Mmm!

Well, that cannot done, so we ‘carry’ and the 2nd decimal place digit increases by one, to 3, and we report the 3rd decimal place as a zero.

124.12954 = 124.130 correct to 3 d.p.

Example 4.

Round 55.00299999 correct to 2 d.p..

0

The digit in the 3rd decimal place is a 2.

2 is less than 5, so the digit in the 2nd decimal place remains the same.

55.00299999 = 55.00 correct to 2 d.p.

Question 1

Round correct to the number of decimal places stated.

a) 54.34952, two d.p.

b) 103.725745, four d.p.

c) 0.9814372, five d.p.

d) 0.21544, three d.p.

e) 23.0499999, one d.p.

f) 54.8434, zero d.p.

Question 2

Round correct to the number of decimal places stated.

a) $0.\overline{6}$, four d.p.

b) $0.\overline{284}$, three d.p.

c) $0.\overline{284}$, four d.p.

d) 48.1195, three d.p.

e) 54.999, two d.p.

f) 0.0004798, three d.p.

Question 3

Ashley thinks that when rounding 27.0248 correct to two decimal places, 27.03 should be the correct result.
Kim disagrees and thinks the answer should be 27.02 but cannot explain why this makes sense.

Kim is correct.

Do your best to explain why it makes sense that 27.02 is correct.
Stating the rule will *not* explain why it makes sense.

Question 4

Earlier it was claimed, without justification, that 3.76 km = 3 km +700 m + 60 m

Do your best to explain why this is true.

Question 5

What is the largest number that when rounded becomes, 32.3 correct to 1 d.p..

23.8 Miscellaneous exercises

Question 1

Jane has six small gold nuggets. The mass of the nuggets are: 25.34 grams, 21.74 grams, 42.61 grams, 21.79 grams, 11.19 grams and 33.45 grams. Jane gives the lightest three to her brother. Did she give away more or less than half of her gold by mass?

Question 2

Lisa decides to sell her 24 ct jewellery. She has a ring that weighs 5.64 grams and a necklace with pendant that weighs 4.36 grams. 24 ct gold sells for $79.68 per gram. What is the total value (in $) of her 24 ct jewellery?

Question 3

The lap record at Anthony's home go kart track is 34.37 seconds. His best time so far is 36.02 seconds. What is the lowest number of seconds by which he must improve to set a new track record?

Question 4

When this question was written, 1 Australian dollar was worth 0.722414 US dollar. Taryn changes 60 Australian dollars to US dollars at the airport. How many US dollars will she get? (Ignore fees.)

Question 5

Roger needs to cut pieces of fuel hose, 0.6 metres long, from a spool that contains 5.6 metres of fuel hose. How many whole 0.6 metres pieces will Roger be able to cut?

Question 6

Oscar is paid $31.9677 per hour. (This is a true value.) If he works 40 hours this week, how much money does he receive? (Ignore tax.)

Question 7

Today I ate 100 grams of Weet-Bix. In doing so I consumed fat, fibre, sugar, … , as well *mineral elements*. The label says I consumed 0.264 grams of sodium, 0.310 grams of potassium, 0.0063 grams of iron and 0.087 grams of magnesium. What was the total mass of the mineral elements I consumed?

Question 8

An injectable vaccine is supplied in small bottles that hold 20 ml. The adult dose of the drug is 0.3 ml. How many whole doses can be drawn from the bottle?

Question 9

A nurse must do a calculation to determine how much of a certain drug to give a child. The drug dose is 0.3 milligrams per kilogram of body weight. The child weighs 25.6 kilograms. The drug is supplied as an oral liquid.
It contains 5 milligrams of the drug per 10 millilitres of liquid.
How much of the liquid should the child take?

Question 10

Write down the next four numbers in each of these patterns.

a) 1.1, 1.21, 1.331, …

b) 11.2, 9.4, 7.6 , ….

c) 0.4, 0.2 , 0.16, 0.04, 0.064, 0.008, …

Question 11

Use your calculator to write each of the following fractions in decimal form.

Do you notice anything interesting about the results?

a) $\frac{1}{7}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) $\frac{4}{7}$ e) $\frac{5}{7}$ f) $\frac{6}{7}$

23.9 Not-so-*nice* calculations - estimate & reach for the calculator

Throughout this series of booklets, you have done a lot of calculations. For example, you calculated,

- a fraction of a quantity,
- a percentage of a quantity,
- what one quantity is as a percentage of another
- …

Most of the calculations you performed were *nice*. Nice in the sense that the numbers were kind, and you were hopefully able to use your mind to do the calculations.
It was important for your development. ☺

However, life does not always deal us kind numbers and when that happens – *reach for the calculator*. ☺

In each of this section's questions, first estimate the answer and then calculate on your calculator. Also write down the calculation you are doing.
Where possible, give your answers as *both* a fraction in simplest form and a number in decimal form rounded correct to 2 decimal places or as appropriate for the context.

Question 1

Calculate:

a) $\frac{3}{7}$ of 219 b) 6% of \$425 c) $34\frac{2}{3}$% of \$30589 d) 125% of \$61.57

Question 2

What percentage is:

a) 15 of 299 b) 2.52 of 10.02 c) 154 of 51 d) 0.045 of 0.0089

Question 3

Apparently, cats spend 66% of their life asleep.
Suppose my cat slept for 66% of a 24-hour period, how many hours did my cat sleep?

Question 4

Apparently, toilet flushing uses 35% of a household's daily water use. If a given household uses 1340 litres per day, how many litres are being flushed?

Question 5

The movie Pulp Fiction cost about $18.5 million to make. $10 million was spent on actors' salaries. What percentage of the cost was spent on actors' salaries?

Question 6

Apparently, 42% of males do not wash their hands after using the toilet! Supposing this is true at a particular all-boys school of 1185 students, how many boys at the school do not wash their hands after using the toilet?

Question 7

Experiments show that when water freezes, it expands by 9.18%. If the 12 spaces in my ice tray are each filled with 85 cubic centimetres of water, calculate the total volume of the ice blocks that result from freezing?

Question 8

Of the 20 000 species of bees that exist on Earth, only 8 species produce honey.
What percentage of all bee species are producers of honey?

Question 9

It was estimated in 2017 that Australian's generated 67 million tonnes of waste. 37 million tonnes were recycled and 21.7 million tonnes were put into landfill. What percentage of the waste generated was put into landfill?

Question 10

Apparently only 2% of the world's humans have green eyes. Calculate 2% of the total number of students in your school. Do you think this many students will have green eyes at your school? If yes, why? If not, why not?

Question 11

The mass of the Earth is thought to be 5.9722×10^{24} kilograms. It is also thought that 32% of the Earth's mass is iron (Fe).

a) How many kilograms of iron exist in/on the Earth?

b) More correctly, the mass of the Earth is thought to be $(5.9722 \pm 0.0036) \times 10^{24}$ kilograms. Calculate a *range* for the number of kilograms of iron we think exists in/on the Earth?

23.10 Recurring things and other mind-blowers

Question 1

Earlier you learned that every fraction has a decimal form that either *terminates* or *recurs*. This question focuses fractions with a decimal form that recurs.

For example, $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$.

Consider the number with decimal form $0.\overline{25}$.

Ari reasoned that,

$$0.\overline{25} = 0.25252525\ldots = \frac{25252525\ldots}{100000000\ldots}$$

Join with a classmate, or three, and explore the task of writing $\frac{25252525\ldots}{100000000\ldots}$ in simplest form.

Question 2

This question follows on from Question 1.

Noah asked whether or not a calculator could convert $0.\overline{25}$ to a fraction in simplest form. It will depend on the type of calculator you have, but Noah typed in 0.252525… until the number of 25s was greater than the calculator's screen could display. He then pressed = followed by the key that converts a number in decimal form to a fraction.
To his (and everyone else's) amazement, the calculator displayed $\frac{25}{99}$.

So, the calculator led Noah to believe that $0.\overline{25} = \frac{25}{99}$, he asked his teacher if that was right.

Noah's teacher then showed the class this:

Let x be the vulgar fraction that is equal to $0.\overline{25}$. So, $x = 0.\overline{25}$. If $x = 0.\overline{25}$ then $100x = 25.\overline{25}$ Now, $100x - x = 99x$ and $25.\overline{25} - 0.\overline{25} = 25$ Therefore, $99x = 25$ So, $x = \frac{25}{99}$

Noah's eyes were the size of saucers.
Use the method used by Noah's teacher to find the vulgar fraction equal to each of the following:

a) $0.\overline{36}$
b) $0.\overline{57}$
c) $0.\overline{75}$
d) $0.\overline{125}$
e) $0.\overline{747}$
f) $0.\overline{4}$
g) $0.\overline{7}$
h) $0.\overline{9}$

Question 3

Were you surprised by the answer to Question 2 part h). 🤯 🙂

Here is an infinite series for you to explore,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$$

Take your time and calculate, carefully, 10 steps of this infinite series.
Write the result of each step in decimal form (and fractional form for as long as your calculator can cope).

Do you notice anything interesting about the results?

Question 4

This is James' infinite series,

$$\frac{1}{4}+\frac{1}{400}+\frac{1}{40000}+\frac{1}{4000000}+\ldots$$

Figure out what number this infinite series adds up to and write the number as both a decimal form that recurs, and a vulgar fraction.

Question 5

This question asks you to explore a different type of infinite thingy.
It is called a continuous fraction and one example is,

$$\cfrac{6}{1+\cfrac{6}{1+\cfrac{6}{1+\cfrac{6}{1+\ \ldots}}}}$$

The first step of this critter is $\frac{6}{1}$. The second step is $\frac{6}{1+\frac{6}{1}}$.

Take your time and calculate, carefully, 10 steps of this infinite series.
Write the result of each step in decimal form (and fractional form for as long as your calculator can cope).

Do you notice anything interesting about the results?

Question 6

Which fractions have decimal forms that terminate? Which fractions have decimals forms that recur?
We will call fractions whose decimal form terminates – *terminators*.
We will call fractions whose decimal form recurs – *recur-ers*.

Fill the columns of the table below with as many fractions as you can.

Terminators	Recur-ers

Do you notice any characteristics possessed by *all* the terminators?
Do you notice any characteristics possessed by *all* the recur-ers?

Question 7

Márta makes the following claim:

If a fraction is in simplest form and it is a *terminator*, then all other fractions with that denominator will be terminators.

Is Márta correct? Justify your answer.

Question 8

Earlier you would have seen that $\frac{1}{7} = 0.14285714\ldots = 0.\overline{142857}$.
The repeating digit sequence contains six digits. The repeating digit sequence is called the *repetend*. In this case the repetend length = six.

Fill in each row of the following table with a different unit fraction that is a *recur-er*.

Fraction	Decimal form	Repetend length	Fraction	Decimal form	Repetend length
$\frac{1}{7}$	$0.\overline{142857}$	6			

Does the data in the table reveal anything interesting?